Many-body quantum physics with nonlinear propagating light

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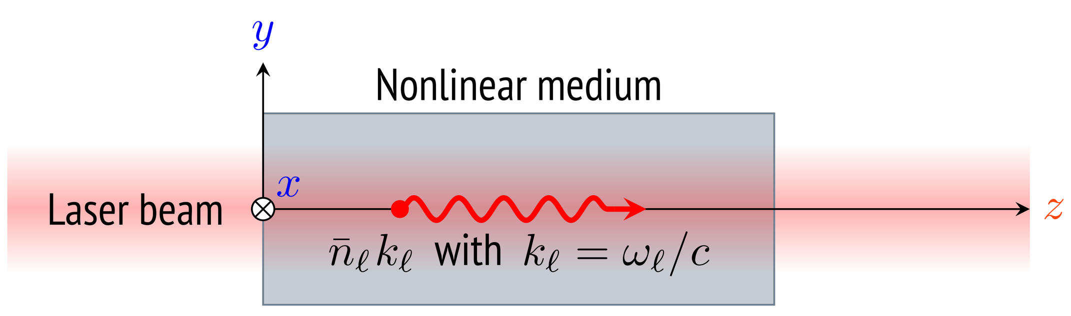
Propagating quantum fluid of light (1)



- Complex scalar electric field of the laser beam:

$$E = \underbrace{\mathcal{E}(x,y,z,t)}_{\text{Envelope}} \times \underbrace{e^{i(\bar{n}_\ell k_\ell z - \omega_\ell t)}}_{\text{Carrier}}$$

$(x,y) \longleftrightarrow (k_x,k_y)$ — $\underbrace{|k_x|,|k_y| \ll k_\ell}_{\text{Paraxiality}}$

$z \longleftrightarrow \Delta k_z = k_z - \bar{n}_\ell k_\ell$

$t \longleftrightarrow \Delta\omega = \omega - \omega_\ell$ — $\underbrace{|\Delta\omega| \ll \omega_\ell}_{\text{Quasimonochromaticity}}$

- Diffraction-dispersion relation and refractive index of the nonlinear medium:

$$\Delta k_z = \underbrace{-\frac{1}{2\,\bar{n}_\ell k_\ell}(k_x^2+k_y^2)}_{\text{Transverse diffraction}} + \underbrace{\frac{\beta_\ell}{2}\Delta\omega^2}_{\text{Chromatic dispersion}} + \cdots$$

$$n = \underbrace{\bar{n}_\ell}_{\text{Mean linear refractive index}} + \underbrace{\Delta n(x,y,z,t)}_{\text{Space-time modulation of the linear refractive index}} + \underbrace{K\,|\mathcal{E}|^2}_{\text{Kerr nonlinear refractive index}} + \cdots$$

- Classical nonlinear Schrödinger theory:

$$\underbrace{i\frac{\partial\mathcal{E}}{\partial z}}_{\text{Longitudinal propagation}} = \underbrace{-\frac{1}{2\,\bar{n}_\ell k_\ell}\left(\frac{\partial^2\mathcal{E}}{\partial x^2}+\frac{\partial^2\mathcal{E}}{\partial y^2}\right)}_{\text{Transverse diffraction}} + \underbrace{\frac{\beta_\ell}{2}\frac{\partial^2\mathcal{E}}{\partial t^2}}_{\text{Chromatic dispersion}} \underbrace{- k_\ell\,\Delta n(x,y,z,t)\,\mathcal{E}}_{\text{Linear refraction}} \underbrace{- k_\ell\,K\,|\mathcal{E}|^2\,\mathcal{E}}_{\text{Nonlinear refraction}}$$

- Forward-propagating envelope: $\Delta k_z > 0$ $\Longrightarrow z \equiv$ Time variable
- Arbitrarily-detuned envelope: $\Delta\omega \lessgtr 0$ $\Longrightarrow t \equiv$ Space variable
- Space-time mapping: $(x,y,z,t) \longmapsto (x,y,t,z)$

- Quantum nonlinear Schrödinger theory from a Dirac quantization at equal z's and different (x,y,t)'s:

$$i\frac{\partial\hat{\mathcal{E}}}{\partial z} = -\frac{1}{2\,\bar{n}_\ell k_\ell}\left(\frac{\partial^2\hat{\mathcal{E}}}{\partial x^2}+\frac{\partial^2\hat{\mathcal{E}}}{\partial y^2}\right) + \frac{\beta_\ell}{2}\frac{\partial^2\hat{\mathcal{E}}}{\partial t^2} - k_\ell\,\Delta n(x,y,t,z)\,\hat{\mathcal{E}} - k_\ell\,K\,\hat{\mathcal{E}}^\dagger\hat{\mathcal{E}}\,\hat{\mathcal{E}}$$

$$[\hat{\mathcal{E}}(x,y,t,z),\hat{\mathcal{E}}^\dagger(x',y',t',z)] = \frac{2\,\hbar\,k_\ell}{\varepsilon_0\,\bar{n}_\ell}\,\delta(x-x')\,\delta(y-y')\,\delta(t-t')$$

- Formal equivalence with the quantum nonlinear Schrödinger theory of dilute atomic Bose gases:

$(x,y,t),\ z \equiv$ Position, Time

$\hat{\mathcal{E}}(x,y,t,z) \equiv$ Matter quantum field

$\mathrm{diag}(\bar{n}_\ell k_\ell, \bar{n}_\ell k_\ell, -1/\beta_\ell) \equiv$ Mass

$-k_\ell\,\Delta n(x,y,t,z) \equiv$ External potential

$-k_\ell\,K \equiv$ Two-body contact-interaction constant

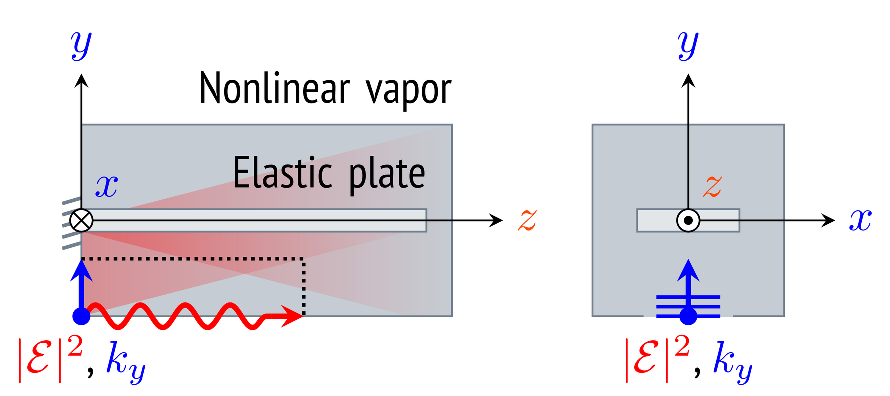
"Propagating quantum fluid of light" $\neq$ Cavity

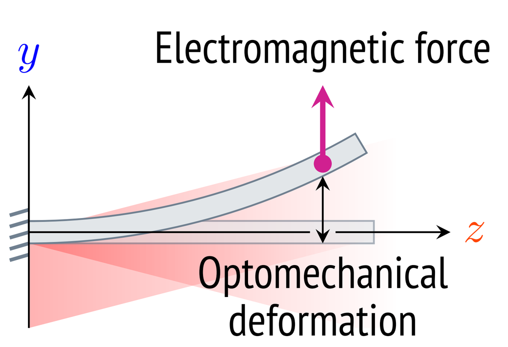
- Quantum nonlinear Schrödinger theory for single-mode nonlinear guides:

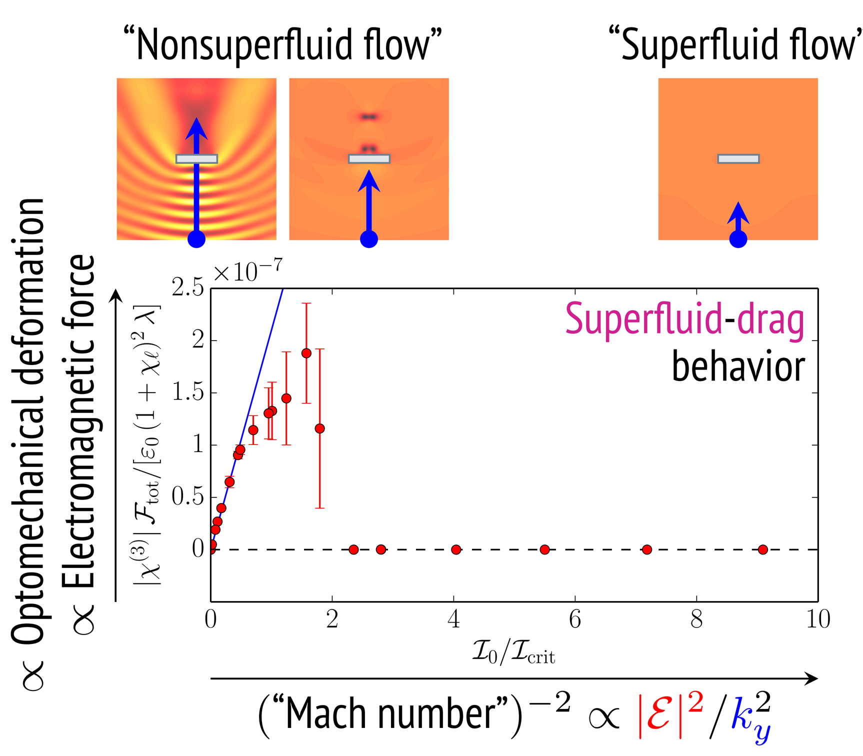
$$i\frac{\partial\hat{\mathcal{E}}_\parallel}{\partial z} = \frac{\beta_\ell}{2}\frac{\partial^2\hat{\mathcal{E}}_\parallel}{\partial t^2} - k_\ell\,\Delta n_\parallel(t,z)\,\hat{\mathcal{E}}_\parallel - k_\ell\,K_\parallel\,\hat{\mathcal{E}}_\parallel^\dagger\,\hat{\mathcal{E}}_\parallel\,\hat{\mathcal{E}}_\parallel$$

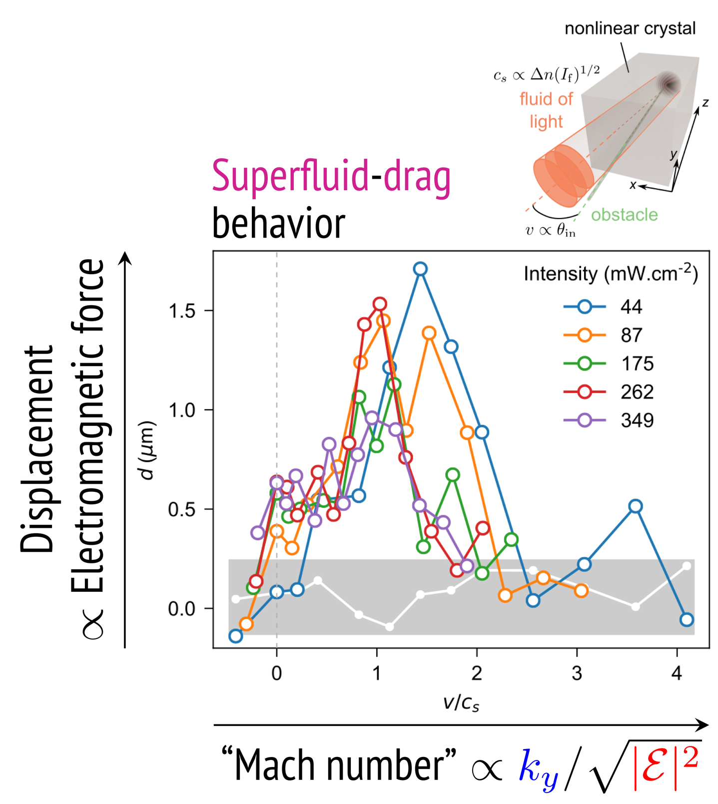
$$[\hat{\mathcal{E}}_\parallel(t,z),\hat{\mathcal{E}}_\parallel^\dagger(t',z)] = \frac{2\,\hbar\,k_\ell}{\varepsilon_0\,\bar{n}_\ell}\,\delta(t-t')$$

Optomechanical and all-optical signatures of a frictionless flow of superfluid light (2, 3)

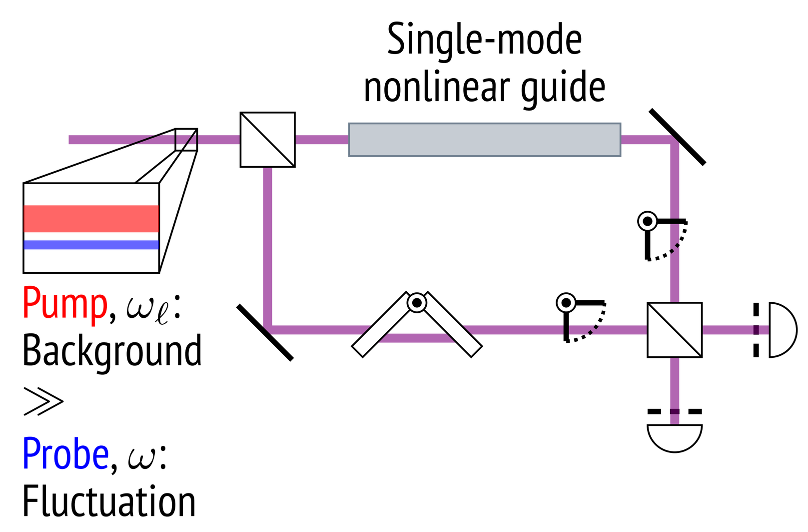






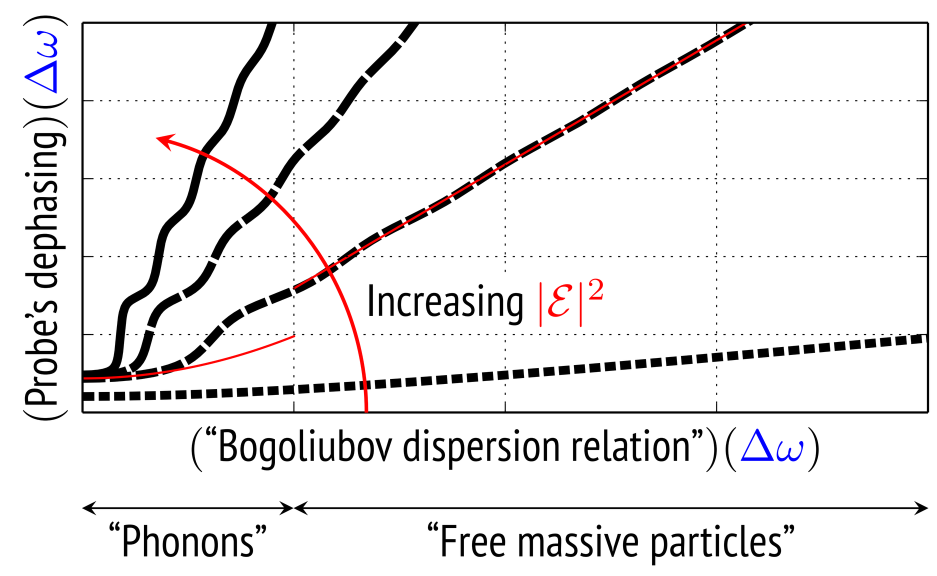


Pump-and-probe interferometry of a Bogoliubov-fluctuating light (4)

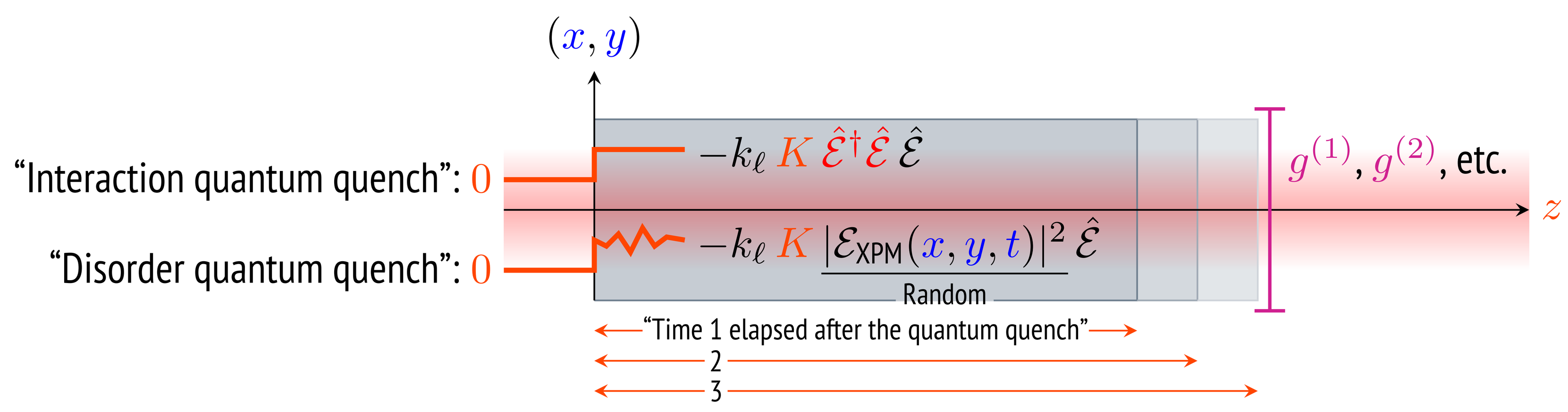


("Bogoliubov dispersion relation")($\Delta\omega$)

$$= \sqrt{\frac{\Delta\omega^2}{2\,(-1/\beta_\ell)}\left[\frac{\Delta\omega^2}{2\,(-1/\beta_\ell)} + 2\,(-k_\ell\,K\,|\mathcal{E}|^2)\right]}$$

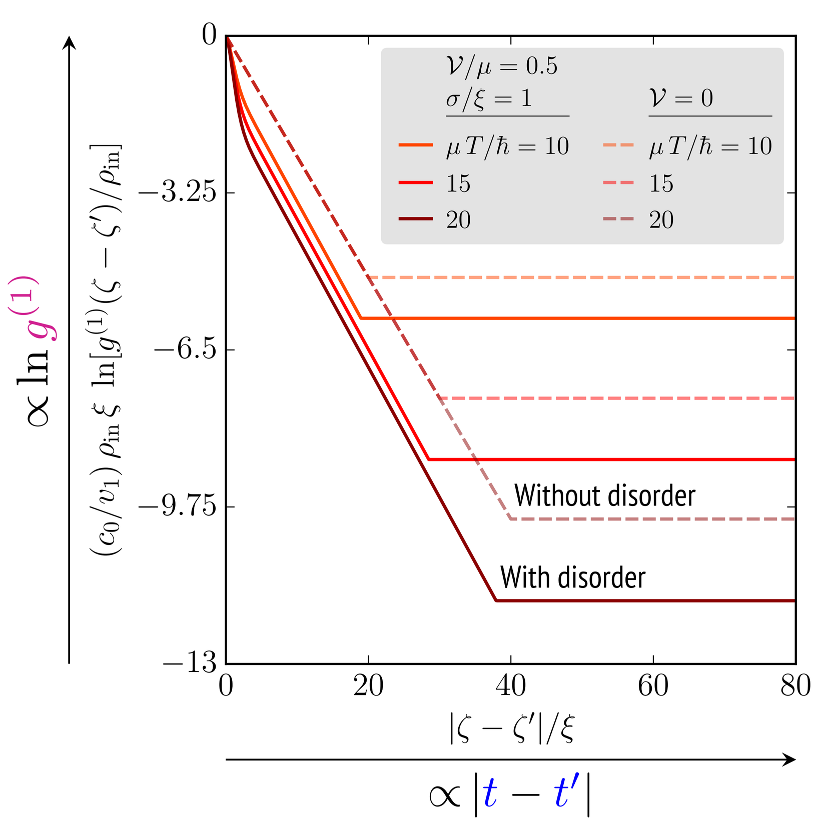


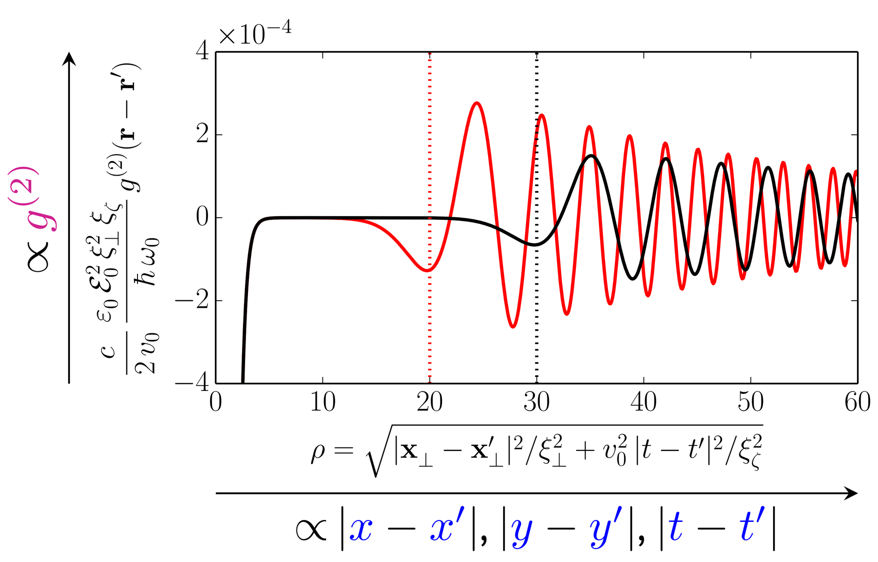
A many-body quantum simulator for quantum-quench physics (1, 5, 6)



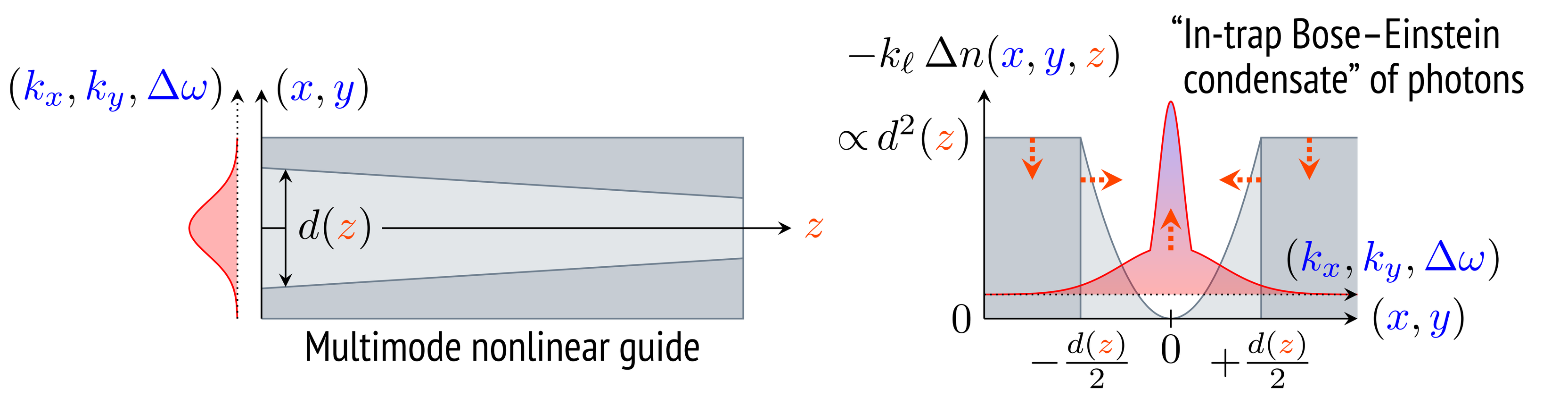
Single-mode nonlinear guide — Light-cone-like "prethermalization" and decoherence

Nonlinear bulk — Light-cone-like spreading of the two-body correlations





Bose–Einstein condensation of photons from an evaporative cooling of incoherent light (7)



("Temperature")(z) $\propto$ (Typical width of the momentum-energy distribution)^2(z)

References

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