Quantum Many-Body Physics with Nonlinear Propagating Light

Pierre-Élie Larré

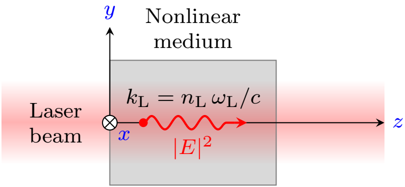
Laboratoire de Physique Théorique et Modélisation

Centre National de la Recherche Scientifique

Université de Cergy-Pontoise

February 6, 2019

Propagation of a Quantum Fluctuating Laser Beam in a Nonlinear Medium



Laser Beam

- Complex electric field:

$$E = \underbrace{\mathcal{E}(x, y, z, t)}_{\text{Envelope}} \times \underbrace{e^{i(k_{\mathrm{L}} z - \omega_{\mathrm{L}} t)}}_{\text{Carrier}}$$

$$(x, y) \longleftrightarrow (k_x, k_y)$$

$$z \longleftrightarrow \Delta k_z = k_z - k_{\mathrm{L}}$$

$$t \longleftrightarrow \Delta\omega = \omega - \omega_{\mathrm{L}}$$

- Paraxiality and quasimonochromaticity:

$$|k_x|, |k_y| \ll k_{\mathrm{L}} \quad \text{and} \quad |\Delta\omega| \ll \omega_{\mathrm{L}}$$

Nonlinear Medium

- Diffraction-dispersion relation in the reference frame of the propagating wave:

$$\Delta k_z = \underbrace{-\frac{1}{2\,k_{\mathrm{L}}}\,(k_x^2 + k_y^2)}_{\text{Diffraction}} + \underbrace{\frac{D_{\mathrm{L}}}{2}\,\Delta\omega^2}_{\text{Dispersion}} + \cdots$$

- Refractive index:

$$n = \underbrace{n_{\mathrm{L}}}_{\substack{\text{Mean } \textbf{linear} \\ \text{refractive index}}} + \overbrace{\Delta n(x, y, z, t)}^{\substack{\text{Space-time modulation of the } \textbf{linear} \\ \text{refractive index}}} + \underbrace{K\,|E|^2}_{\substack{\text{Kerr } \textbf{nonlinear} \\ \text{refractive index}}} + \cdots$$

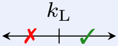
$(3+1)$D Optical Nonlinear Schrödinger Equation

In the reference frame of the propagating wave:

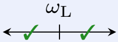
$$\underbrace{i\,\frac{\partial \mathcal{E}}{\partial z}}_{\text{Propagation}} = \underbrace{-\frac{1}{2\,k_{\mathrm{L}}}\left(\frac{\partial^2 \mathcal{E}}{\partial x^2}+\frac{\partial^2 \mathcal{E}}{\partial y^2}\right)}_{\text{Diffraction}} + \underbrace{\frac{D_{\mathrm{L}}}{2}\,\frac{\partial^2 \mathcal{E}}{\partial t^2}}_{\text{Dispersion}} \underbrace{-\,k_{\mathrm{L}}\,\frac{\Delta n(x,y,z,t)}{n_{\mathrm{L}}}\,\mathcal{E}}_{\textbf{Linear}\text{ refraction}} \underbrace{-\,k_{\mathrm{L}}\,\frac{K\,|\mathcal{E}|^2}{n_{\mathrm{L}}}\,\mathcal{E}}_{\textbf{Nonlinear}\text{ refraction}}$$

"Propagating Quantum Fluid of Light"

- Space $\longleftrightarrow$ Time mapping:



$\Delta k_z \in (0,+\infty)$
$\Longrightarrow z \equiv$ Time variable



$\Delta \omega \in (-\infty,+\infty)$
$\Longrightarrow t \equiv$ Space variable

- Dirac quantization at equal z's and different (x,y,t)'s:

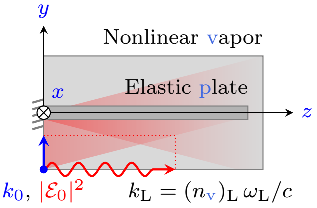
$$i\,\frac{\partial \hat{\mathcal{E}}}{\partial z} = -\frac{1}{2\,k_{\mathrm{L}}}\left(\frac{\partial^2 \hat{\mathcal{E}}}{\partial x^2}+\frac{\partial^2 \hat{\mathcal{E}}}{\partial y^2}\right) + \frac{D_{\mathrm{L}}}{2}\,\frac{\partial^2 \hat{\mathcal{E}}}{\partial t^2} - k_{\mathrm{L}}\,\frac{\Delta n(x,y,t,z)}{n_{\mathrm{L}}}\,\hat{\mathcal{E}} - k_{\mathrm{L}}\,\frac{K\,\hat{\mathcal{E}}^\dagger\hat{\mathcal{E}}}{n_{\mathrm{L}}}\,\hat{\mathcal{E}}$$

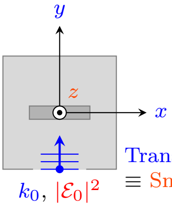
$$[\hat{\mathcal{E}}(x,y,t,z),\hat{\mathcal{E}}^\dagger(x',y',t',z)] = \frac{2\,\hbar\,k_{\mathrm{L}}}{\varepsilon_0\,(n_{\mathrm{L}})^2}\,\delta(x-x')\,\delta(y-y')\,\delta(t-t')$$

P.-É. Larré and I. Carusotto, Phys. Rev. A **92**, 043802 (2015)

- Formal analogy with the quantum many-body theory of dilute atomic Bose gases

Optomechanical Signature of a Frictionless Flow of Superfluid Light





Transverse slice
≡ Snapshot

P.-É. Larré and I. Carusotto,
Phys. Rev. A **91**, 053809 (2015)

$\Longrightarrow$ In-progress experiment @
Laboratoire Kastler–Brossel (Paris)

Effective 2D Gross–Pitaevskii Theory

- Plate $\Longrightarrow$ Local modification of the linear refractive index of the vapor:

$$\Delta n(x,y) = \begin{cases} (n_\mathrm{p})_\mathrm{L} - (n_\mathrm{v})_\mathrm{L}, & (x,y) \in \text{plate} \\ 0, & (x,y) \in \text{vapor} \end{cases}$$

- In the mean-field and monochromatic approximations:

$$i\,\frac{\partial \mathcal{E}}{\partial z} = -\frac{1}{2\,k_\mathrm{L}}\left(\frac{\partial^2 \mathcal{E}}{\partial x^2} + \frac{\partial^2 \mathcal{E}}{\partial y^2}\right) - k_\mathrm{L}\,\frac{\Delta n(x,y)}{(n_\mathrm{v})_\mathrm{L}}\,\mathcal{E} - k_\mathrm{L}\,\frac{K\,|\mathcal{E}|^2}{(n_\mathrm{v})_\mathrm{L}}\,\mathcal{E}$$

Effective Flow

- Initial condition far upstream from the obstacle:

$$\mathcal{E}(x, y \to -\infty, z = 0) = \sqrt{|\mathcal{E}_0|^2}\, e^{i k_0 y}$$

- Associated Mach number:

$$M = \frac{\text{Flow speed} = \dfrac{k_0}{k_\mathrm{L}} \simeq \text{Incidence angle}}{\text{Sound speed} = \sqrt{-k_\mathrm{L}\,\dfrac{K\,|\mathcal{E}_0|^2}{(n_\mathrm{v})_\mathrm{L}}\Big/ k_\mathrm{L}}}$$

Effective Drag Force

- Electromagnetic-force density experienced by the plate/vapor:

$$\mathbf{f}_{\mathrm{p/v}} = (\mathbf{P}_{\mathrm{p/v}} \cdot \nabla)\,\mathbf{E} + \frac{\partial \mathbf{P}_{\mathrm{p/v}}}{\partial t} \times \mathbf{B}$$

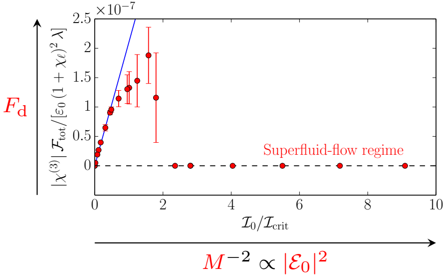
- Electromagnetic force experienced by the plate immersed in the vapor:

$$\mathbf{F} = \iiint \mathrm{d}V_{\mathrm{p}}\, \mathbf{f}_{\mathrm{p}} + \iint \mathrm{d}\mathbf{S}_{\mathrm{v}\to\mathrm{p}}\, \Pi_{\mathrm{v}}$$

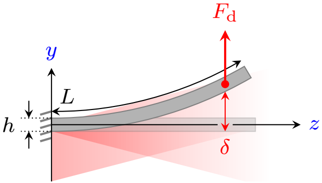
$$\nabla \Pi_{\mathrm{v}} = \mathbf{f}_{\mathrm{v}}$$

- After averaging over a few $2\pi/\omega_{\mathrm{L}}$'s, the corresponding electromagnetic pressure presents a drag-force behavior:

$$F_{\mathrm{d}} \propto \int \mathrm{d}y\, |\mathcal{E}(x,y,z)|^2\, \frac{\partial}{\partial y}\left[-k_{\mathrm{L}} \frac{\Delta n(x,y)}{(n_{\mathrm{v}})_{\mathrm{L}}}\right]$$



Large M: $F_{\mathrm{d}} \neq 0 \iff$ Nonsuperfluid
Low M: $F_{\mathrm{d}} = 0 \iff$ Superfluid



Optomechanical Deformation

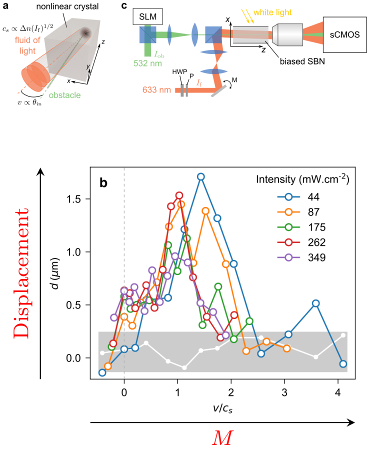
$$\frac{\delta}{L} = (1 - \text{Poisson's ratio}^2)\,\frac{F_d}{\text{Young's modulus}} \times \frac{L^3}{h^3}\left(\frac{1}{2}\frac{z^4}{L^4} - 2\frac{z^3}{L^3} + 3\frac{z^2}{L^2}\right)$$

- Principle of the experiment:

$$\delta = 0 \Longleftrightarrow F_d = 0 \Longleftrightarrow \text{Superfluid}$$

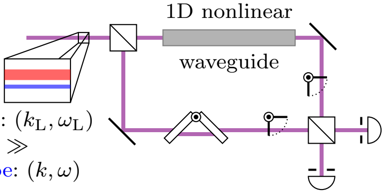
- Fused silica in iodine-doped methanol:

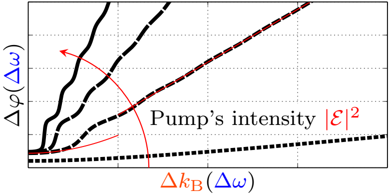
$$F_d \sim 1\,\text{nN/mm}^2 \quad \text{and} \quad \delta \sim 1\,\mu\text{m}$$



C. Michel, O. Boughdad, M. Albert, P.-É. Larré, and M. Bellec, Nat. Commun. **9**, 2108 (2018)

Pump-and-Probe Interferometry of a Bogoliubov-Fluctuating Light





- Effective Bogoliubov dispersion relation:

$$\underbrace{\Delta k_B}_{\sim k - k_L \,\equiv\, \text{Bog. energy}} \quad \text{versus} \quad \underbrace{\Delta\omega}_{= \omega - \omega_L \,\equiv\, \text{Bog. wavenumber}} = \sqrt{\frac{\Delta\omega^2}{2\,(-1/D_L)}\left[\frac{\Delta\omega^2}{2\,(-1/D_L)} + 2\left(-k_L\,\frac{K\,|\mathcal{E}|^2}{n_L}\right)\right]}$$

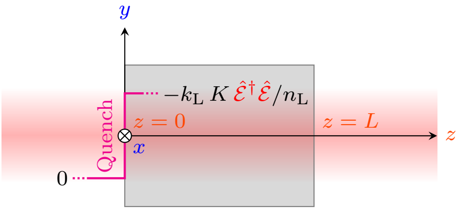
- Probe's dephasing:

$$\Delta\varphi(\Delta\omega) \simeq \begin{cases} \alpha + \beta\,\Delta k_B(\Delta\omega)^2 & \text{at low } \Delta\omega \quad (\text{"phonon" regime}) \\ \gamma\,\Delta k_B(\Delta\omega) & \text{at large } \Delta\omega \quad (\text{"free-particle" regime}) \end{cases}$$

F. Ramiro-Manzano *et al.*, MRS Advances **1**, 3281 (2016)
P.-É. Larré, S. Biasi, F. Ramiro-Manzano, L. Pavesi, and I. Carusotto, Eur. Phys. J. D **71**, 146 (2017)
S. Biasi *et al.*, IEEE Photon. J. **Early Access**, 10.1109/JPHOT.2018.2880281 (2018)

$\Longrightarrow$ In-progress experiments @
Laboratorio di Nanoscienze (Trento)
Laboratoire Kastler–Brossel (Paris)

Nonequilibrium Quantum Many-Body Dynamics after a Quench



P.-É. Larré and I. Carusotto, Phys. Rev. A **92**, 043802 (2015)

P.-É. Larré and I. Carusotto, Eur. Phys. J. D **70**, 45 (2016)

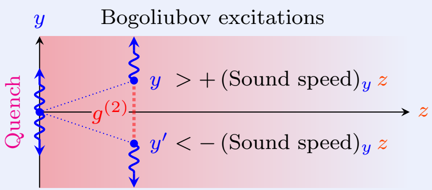
P.-É. Larré, D. Delande, and N. Cherroret, Phys. Rev. A **97**, 043805 (2018)

G. I. Martone, P.-É. Larré, A. Fabbri, and N. Pavloff, Phys. Rev. A **98**, 063617 (2018)

Effective Interaction Quench

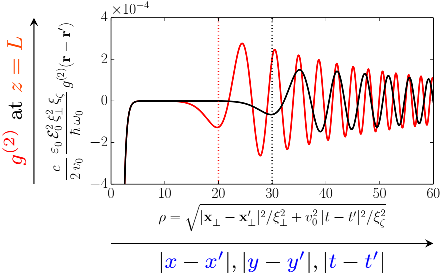
- At $z = 0$, the propagating quantum fluid of light experiences an interaction quench.
- The propagation distance $0 < z < L$ across the medium plays the role of the time elapsed after the quench.
- At $z = L$, by measuring the quantum statistical properties of the transmitted light, one gains insight into the nonequilibrium quantum many-body dynamics of the system.

Light-Cone Effect



$|x - x'|, |y - y'|, |t - t'|$

$> 2\,(\text{Sound speed})_{x,y,t}\; z$

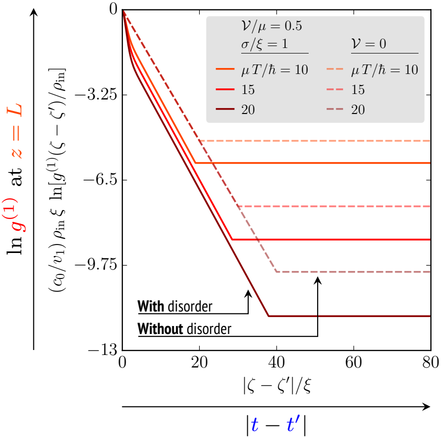


(Pre)thermalization

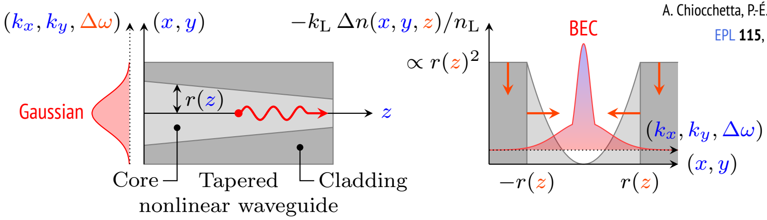
1D nonlinear waveguide, $\mathcal{E} \simeq \underbrace{F(x, y)}_{\text{Frozen}} A(t, z)$:

$$\ln g^{(1)} \propto \begin{cases} 0, & z = 0 \\ -k_{\mathrm{B}}\, T_{\mathrm{eff}}\, |t - t'|, & z \to +\infty \end{cases}$$

$$k_{\mathrm{B}}\, T_{\mathrm{eff}} = \tfrac{1}{2}\,(-k_{\mathrm{L}}\, K\, |\mathcal{E}|^2 / n_{\mathrm{L}})$$



Bose–Einstein Condensation of Photons from an Evaporative Cooling of Incoherent Light



A. Chiocchetta, P.-É. Larré, and I. Carusotto, EPL **115**, 24002 (2016)

Reaching Quantum Degeneracy

- (Effective temperature)$(z) \propto$ (Typical width of the Fourier distribution)$(z)^2$
- Effective evaporative cooling and Bose–Einstein condensation:

z increases $\Longrightarrow$ $r(z)$ decreases

$\Longrightarrow$ The maximum amplitude of the trapping potential decreases

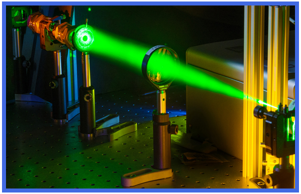
$\Longrightarrow$ | The tails of the Fourier distribution rarefy
$\Longleftrightarrow$ The system cools down

$\Longrightarrow$ $\exists\, z_{\text{crit}},\ \forall\, z > z_{\text{crit}},$ Bose–Einstein condensate

Quantum Many-Body Physics with Nonlinear Propagating Light

The propagation of a laser beam in a nonlinear medium can serve as an analog quantum simulator for quantum many-body dynamics:

- Superfluid hydrodynamics
- Elementary excitations
- Quantum quenches
- Disorder
- Thermalization
- Bose–Einstein condensation
- In-progress: Nonlinear topology
Strongly interacting regime



D. Faccio's "Extreme Light" team (Edinburgh)