Sonic black holes and Hawking radiation in BECs

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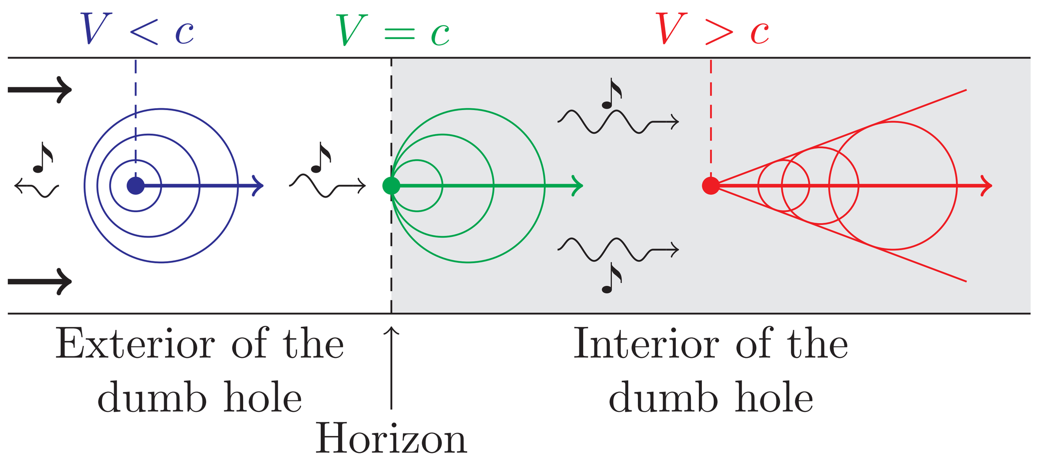
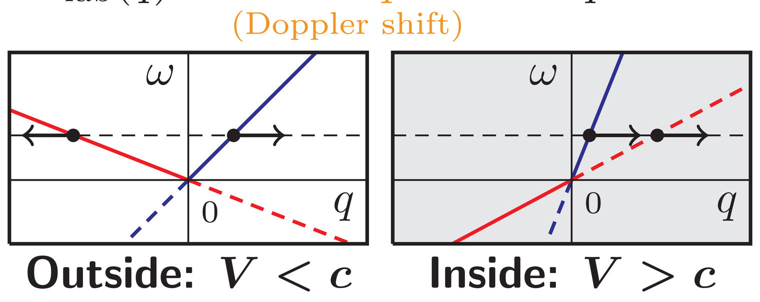
Acoustic black holes

The acoustic black hole is to sound what the gravitational black hole is to light.



$\mathscr{E}_{\rm lab}(q) = V\hbar q$ (Doppler shift) $\pm c\hbar q = \hbar\omega$



Stimulated/**Spontaneous** Hawking radiation.

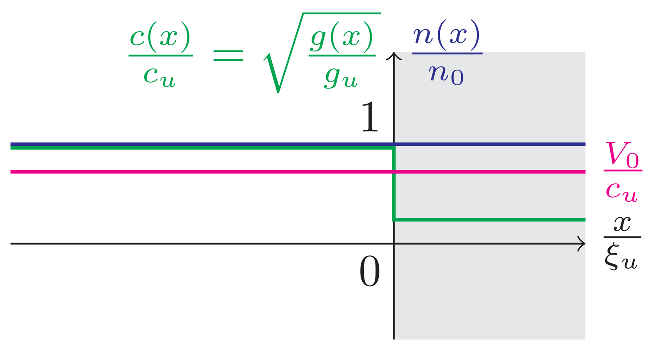
Dumb holes in quasi-1D Bose–Einstein condensates

Stationary Gross–Pitaevskii equation:

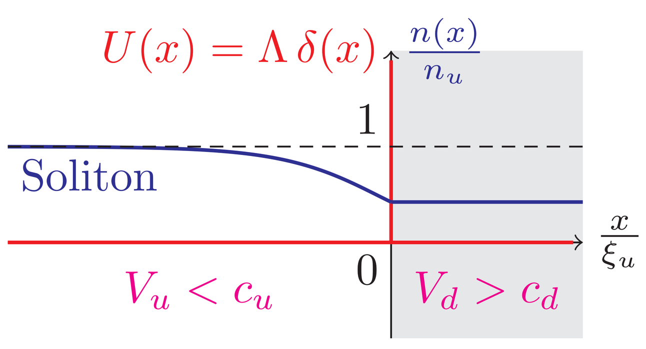
$$\mu\,\Psi = -\frac{\hbar^2}{2m}\,\partial_{xx}\Psi + \big[U(x) + g(x)\,n\big]\Psi.$$

- $\Psi(x)$: BEC order parameter,
- $n(x) = |\Psi(x)|^2$: density,
- $U(x)$: external potential,
- $g(x)$: contact-interaction constant,
- μ: chemical potential.

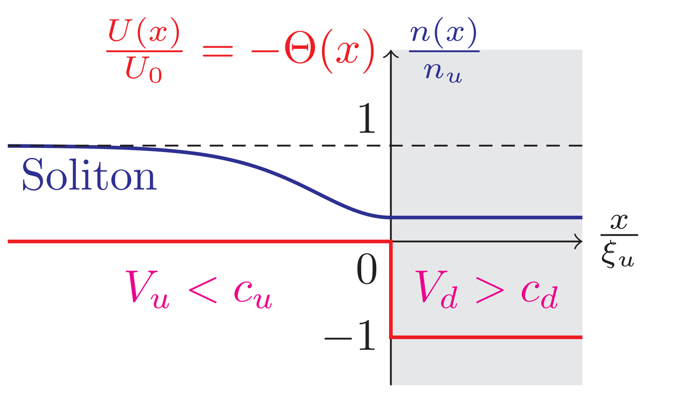
Flat-profile configuration: $g(x)$



δ-peak configuration: $g(x) = C^{\rm st}$



Waterfall configuration: $g(x) = C^{\rm st}$



Quantum fluctuations: Bogoliubov approach

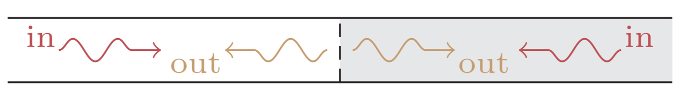
- Bogoliubov approach:

$$\hat\Psi(x,t) = \Psi(x) + \hat\psi(x,t) \quad \text{with} \quad \hat\psi \ll \Psi.$$

- Bogoliubov spectrum:

$$\mathscr{E}_{\rm lab}(q) = V\hbar q\ (\text{Doppler shift}) \pm \mathscr{E}_{\rm B}(q) = \hbar\omega,$$

$$\mathscr{E}_{\rm B}(q) = c\,\hbar\,q\,\sqrt{1 + \frac{\xi^2 q^2}{4}}.$$

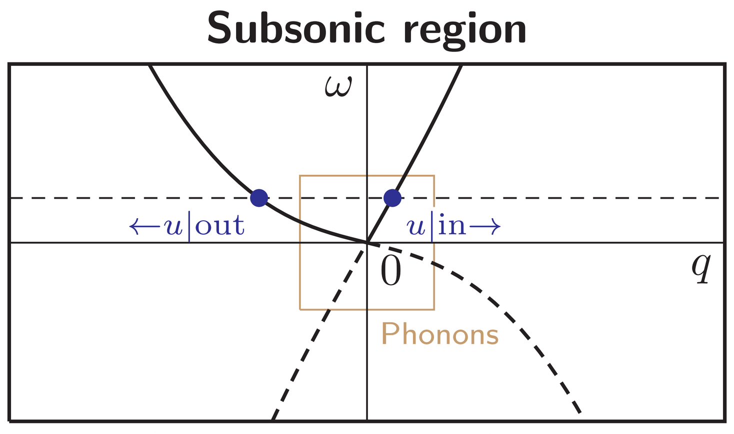


- Scattering matrix:

$$\begin{bmatrix} u|\text{out} \\ d1|\text{out} \\ (d2|\text{out})^\dagger \end{bmatrix} = \mathbf{S}(\omega)\begin{bmatrix} u|\text{in} \\ d1|\text{in} \\ (d2|\text{in})^\dagger \end{bmatrix}.$$

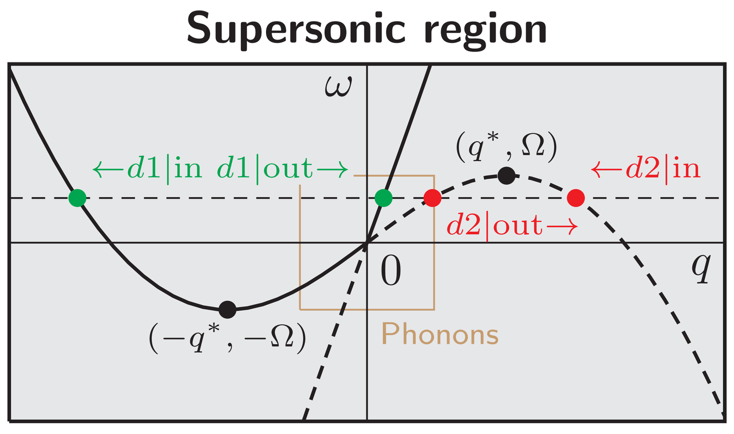
$|\mathbf{S}_{\ell,\ell'}(\omega)|^2$: transmission or reflection coefficient for a ℓ'-ingoing mode oscillating at pulsation ω scatters into a ℓ-outgoing mode.

Subsonic region



$\ell \in \{u|\text{in}, u|\text{out}, u|\text{eva}\}$, $\forall\omega > 0$

Supersonic region



$\ell \in \{d1|\text{in}, d1|\text{out}, d2|\text{in}, d2|\text{out}\}$, $\forall\omega < \Omega$
$\ell \in \{d1|\text{in}, d1|\text{out}, d|\text{eva}\}$, $\forall\omega > \Omega$

One-body Hawking signal

- Energy current associated to the emission of elementary excitations (deep outside the black hole):

$$\Pi_0 \overset{\rm def.}{=} \langle\hat\Pi\rangle_{T=0} = -\int_0^\Omega \frac{d\omega}{2\pi}\,\hbar\omega\,|\mathbf{S}_{u,d2}(\omega)|^2.$$

- Radiation spectrum:

$$|\mathbf{S}_{u,d2}(\omega)|^2 \simeq \frac{\Gamma}{\exp(\hbar\omega/T_{\rm H}) - 1}.$$

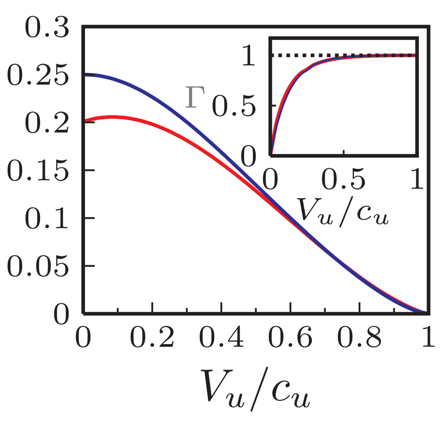
- Low-ω behaviour of $\mathbf{S}_{u,d2}$:

$$\mathbf{S}_{u,d2}(\omega) \simeq f_{u,d2}\left(\frac{\hbar\omega}{m c_u^2}\right)^{-\frac12} + h_{u,d2}\left(\frac{\hbar\omega}{m c_u^2}\right)^{\frac12}.$$

$\Longrightarrow$ Analytical estimates of the gray-body factor and of the Hawking temperature:

$$\Gamma = -4\,\mathrm{Re}(f^*_{u,d2}\,h_{u,d2}), \qquad \frac{T_{\rm H}}{m c_u^2} = \frac{|f_{u,d2}|^2}{\Gamma}.$$

$T_{\rm H} \sim 10\,\text{nK} \ < \ \mu \sim 100\,\text{nK}$

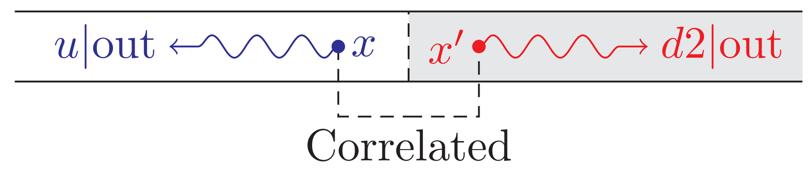


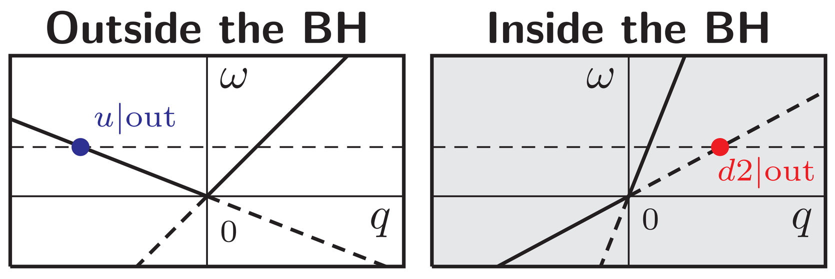
Two-body Hawking signal

- Connected two-body density matrix:

$$g^{(2)}(x,x') = \langle\hat\Psi^\dagger(x,t)\,\hat\Psi^\dagger(x',t)\,\hat\Psi(x,t)\,\hat\Psi(x',t)\rangle - n(x)\,n(x') \overset{\rm def.}{=} n(x)\,n(x')\,G^{(2)}(x,x').$$

Correlated phonons





- At time t after their emission, the phonons $u|$out and $d2|$out are respectively located at

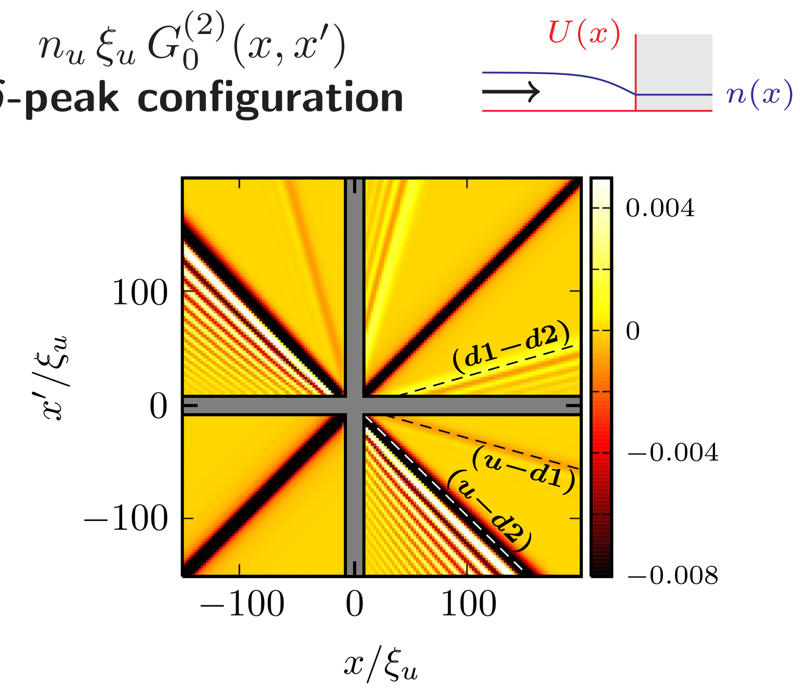
$$x = V_{\rm g}(q_{u|\rm out})\,t \quad \text{and} \quad x' = V_{\rm g}(q_{d2|\rm out})\,t,$$

inducing a correlation signal $(u-d2)$ along the line of slope

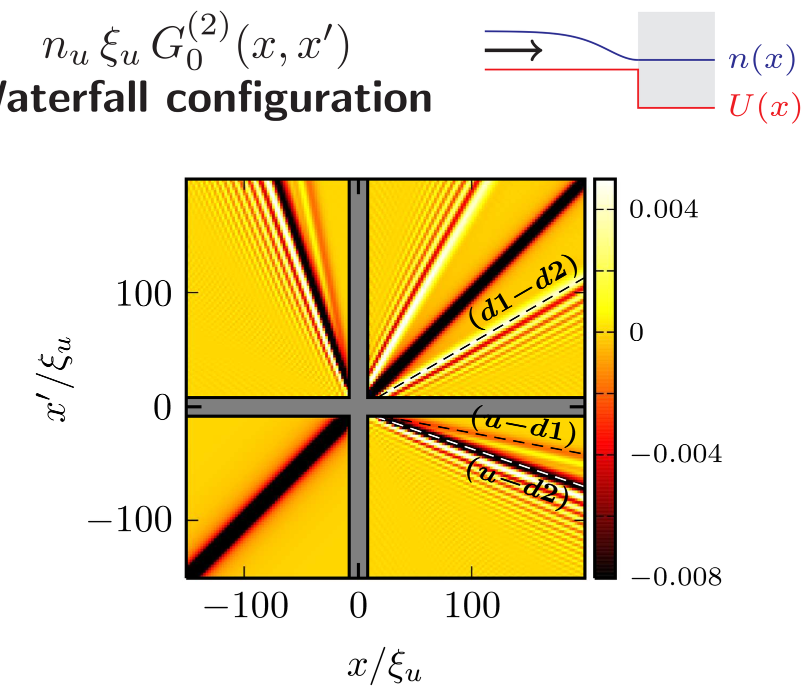
$$\frac{x'}{x} = \frac{V_{\rm g}(q_{d2|\rm out})}{V_{\rm g}(q_{u|\rm out})}$$

in the $\{x,x'\}$ plane.

$n_u\,\xi_u\,G_0^{(2)}(x,x')$
δ-peak configuration



$n_u\,\xi_u\,G_0^{(2)}(x,x')$
Waterfall configuration



Compressibility sum rule at zero temperature

- In the absence of black hole,

$$\int_{\mathbb{R}} dx'\,g_0^{(2)}(x,x') = -n(x).$$

- In the presence of black hole, the shape of the short-range antibunching is modified:

$$\int_{\mathbb{R}} dx'\,g_0^{(2)}(x,x') \leftrightarrow -n(x) + (\text{terms})_{\rm BH}.$$

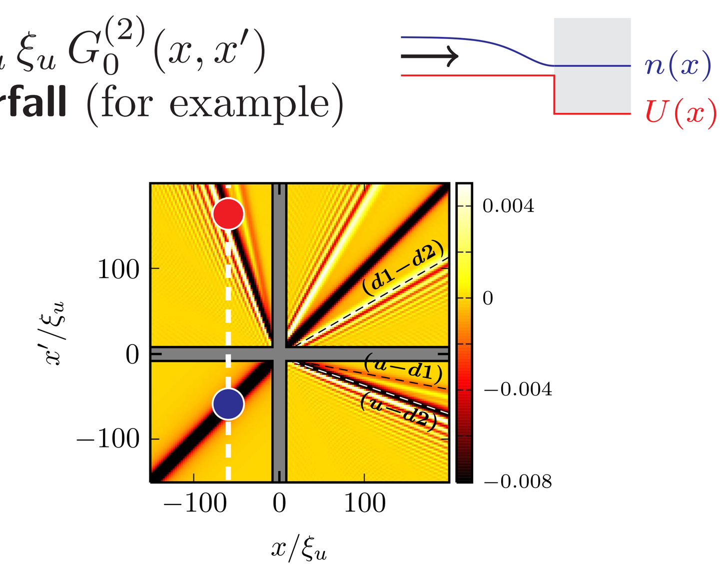
Long-range correlations allow us to recover the sum rule:

$$\int_{\mathbb{R}} dx'\,g_0^{(2)}(x,x') \leftrightarrow -(\text{terms})_{\rm BH}.$$

Because of the sum rule,

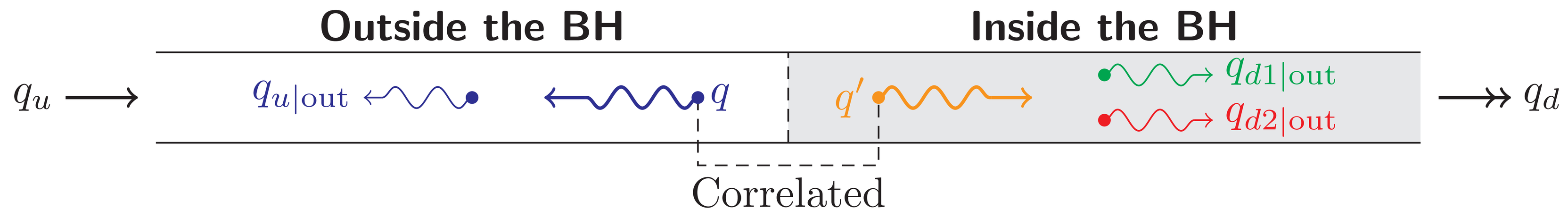
(Long-range correlations)
$\longleftrightarrow$
(Modifications of short-range correlations).

$n_u\,\xi_u\,G_0^{(2)}(x,x')$
Waterfall (for example)

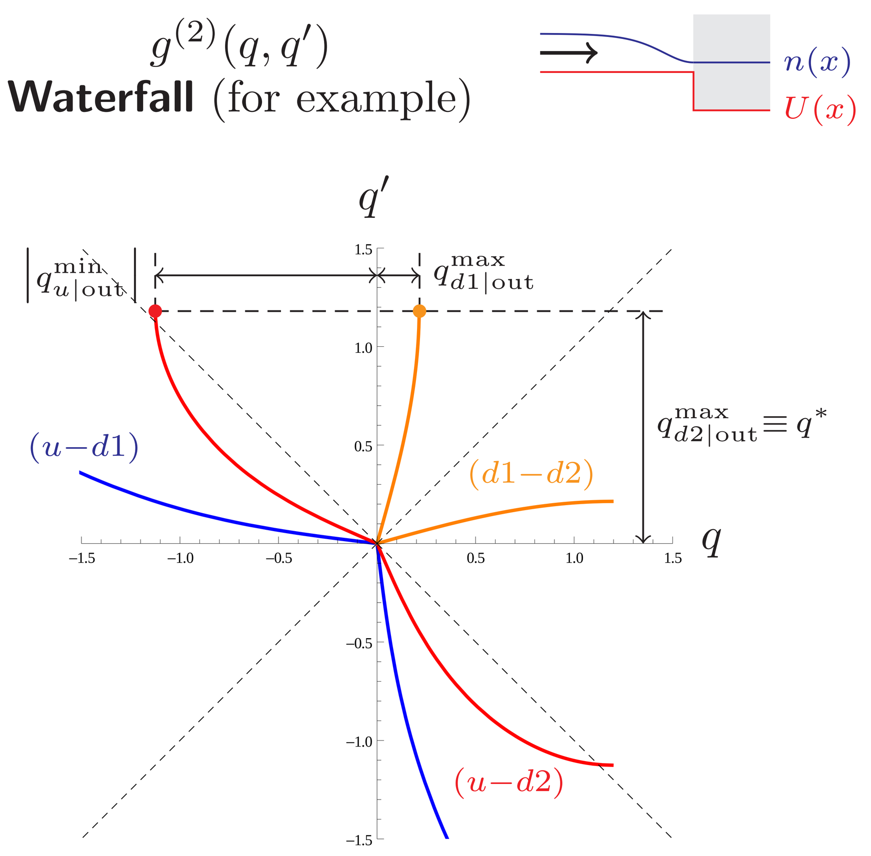


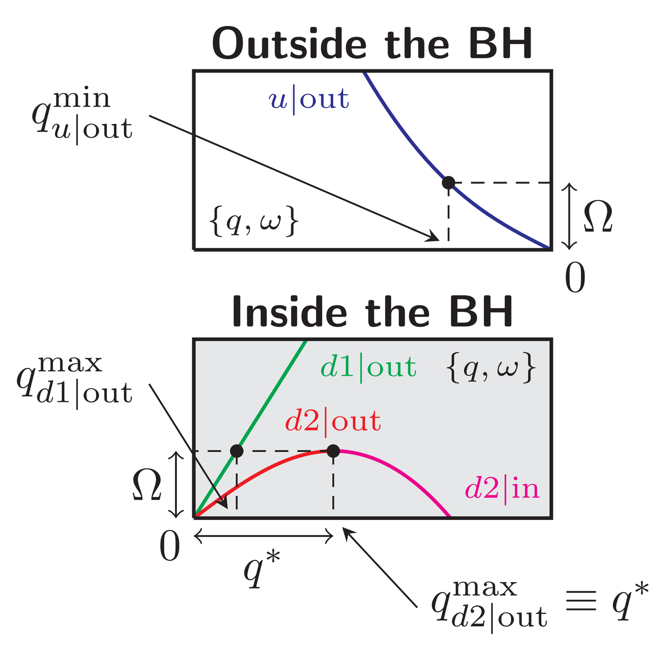
- $\int_{\mathbb{R}} dx'\,g_0^{(2)}(x,x') \xrightarrow[x\to-\infty]{} -n_u + \frac{n_u}{2}\sqrt{\frac{c_u}{c_d}\frac{n_d}{n_u}}\,\mathrm{Re}\left(\frac{f^*_{u,d2}}{1-m_u}\,\mathscr{F}\right)$
- $\int_{\mathbb{R}} dx'\,g_0^{(2)}(x,x') \xrightarrow[x\to+\infty]{} -n_d + \frac{n_d}{2}\left(\frac{c_u}{c_d}\right)^2\mathrm{Re}\left[\left(\frac{f^*_{d1,d2}}{m_d+1} + \frac{f^*_{d2,d2}}{m_d-1}\right)\mathscr{F}\right]$
- $\mathscr{F} = f_{u,d2}\sqrt{\frac{c_d}{c_u}\frac{n_u}{n_d}} + f_{d1,d2} + f_{d2,d2} = 0$
- $m_\alpha \overset{\rm def.}{=} V_\alpha/c_\alpha$ $(\alpha = u, d)$

Two-body Hawking signal in momentum space



$g^{(2)}(q,q')$
Waterfall (for example)





$$\xi_d\,q^* = \left(-2 + \frac{m_d^2}{2} + \frac{m_d}{2}\sqrt{8 + m_d^2}\right)^{\frac12}$$

Ref.: P.-É. L. *et al.*, *Physical Review A* **85**, 013621 (2012) (PhD publication)