

Collective dynamics in binary superfluids: From dissipationless flow to dispersive shock waves

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anr®

1. Introduction

Binary superfluids...

- BEC of interacting atoms in 2 hyperfine spin states ·
polaritons excitonic
Elliptically polarized laser in a birefringent nonlinear medium · ...
Proc. Internat. School Phys. Enrico Fermi **211** (2025)
Rev. Mod. Phys. **85**, 299 (2013)
Adv. At. Mol. Opt. Phys. **74**, 157 (2025)
- $\mathcal{H} = \frac{1}{2}(|\nabla\psi_+|^2 + |\nabla\psi_-|^2) + \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \frac{\alpha|\psi_+|^2|\psi_-|^2}{\text{Inter-component int.}}$ $\alpha = g_{+-}/g_{\pm\pm}$
Intra-component int.
- $\psi_{\pm} = \sqrt{\rho_{\pm}} e^{i \int \mathbf{dr} \cdot \mathbf{v}_{\pm}}$
Density mode $\left\{ \begin{array}{l} \rho = \rho_+ + \rho_- \\ \rho, \mathbf{V} \text{ fluct.} \end{array} \right.$ Spin mode $\left\{ \begin{array}{l} \sigma = \rho_+ - \rho_- \\ \sigma, \mathbf{v} \text{ fluct.} \end{array} \right.$
 $\left\{ \begin{array}{l} \mathbf{V} = \frac{1}{2}(\mathbf{v}_+ + \mathbf{v}_-) \\ \mathbf{v} = \mathbf{v}_+ - \mathbf{v}_- \end{array} \right.$
- Miscibility problems · Collective & topological excitations · ...

...of light

Phys. Rev. Lett. **134**, 223403 (2025)

Elliptically polarized laser in a warm vapor of ^{87}Rb
@ LKB, Paris



Quentin Glorieux



Claire Michel



Nicolas Cherroret

...of matter

Phys. Rev. Lett. **128**, 083401 (2022)

BEC of ^{39}K in 2 coherently coupled Zeeman states
@ LCF, Palaiseau



Thomas Bourdel

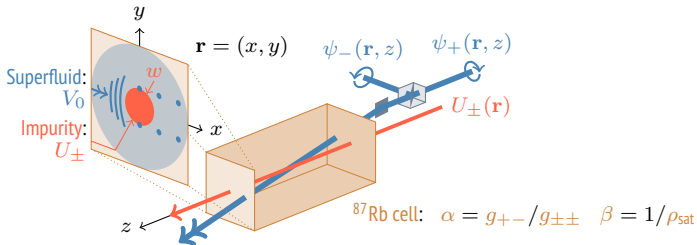


Thibault Congy



Patrick Sprenger

2. Critical speed of a binary superfluid of light



Model equations

- 2-component inhomogeneous nonlinear Schrödinger-type equation, analogous to the Gross-Pitaevskii equation of Bose-Bose superfluid mixtures:

$$i\partial_z \psi_\pm = \left[-\frac{1}{2} \nabla^2 + U_\pm(\mathbf{r}) + \frac{|\psi_\pm|^2 + \alpha |\psi_\mp|^2}{1 + \beta (|\psi_+|^2 + |\psi_-|^2)} \right] \psi_\pm$$

- What is the condition on V_0 for the following incident flow to be superfluid/dissipationless?

$$\psi_\pm^{(\text{in})} = \sqrt{\frac{1}{2}} e^{i(V_0 x - \mu z)}$$

Fully balanced

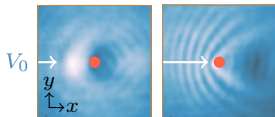
$$\frac{U_\pm > 0}{\text{Obstacle}}$$

$$\frac{g_{\pm\pm} > g_{+-} > 0}{\text{Miscible, repulsive}} \implies 0 < \alpha < 1$$

Linear response

- Impurity $\ll$ Interactions
- Impurity-induced excitations:
Density & spin Bogoliubov waves

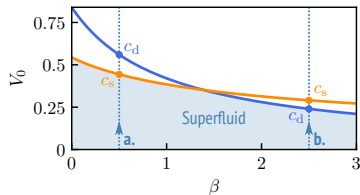
Single-component superfluid of light
in a photorefractive crystal



Nat. Commun. 9, 2108 (2018)

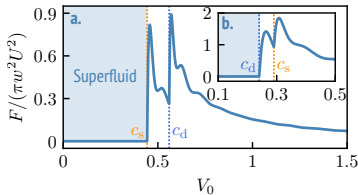
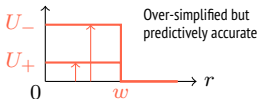
Landau criterion

$$V_0 < V_c = \min \left\{ c_d = \sqrt{\frac{1+\alpha}{2}} \frac{1}{1+\beta}, c_s = \sqrt{\frac{1-\alpha}{2}} \frac{1}{\sqrt{1+\beta}} \right\}$$



Drag force

$$F = \int d^2\mathbf{r} \begin{bmatrix} \psi_+^* & \psi_-^* \end{bmatrix} \begin{bmatrix} \partial_x U_+(\mathbf{r}) & 0 \\ 0 & \partial_x U_-(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$$



Superfluid hydrodynamics

- Stationary Bernoulli equations:

$$\cos \theta \equiv \sigma / \rho \quad U, u \equiv U_+ \pm U_-$$

$$\begin{cases} \frac{(1 + \alpha)\rho}{1 + \beta\rho} = \frac{1 + \alpha}{1 + \beta} - U \mathbf{1}_{r < w} - (V^2 - V_0^2) - \frac{v^2}{4} + \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \tan \theta} - \frac{(\nabla \theta)^2}{4} \\ \frac{(1 - \alpha)\sigma}{1 + \beta\rho} = -u \mathbf{1}_{r < w} - \mathbf{V} \cdot \mathbf{v} - \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \sin \theta} \end{cases}$$

- Stationary continuity equations:

$$\begin{cases} \nabla \cdot \left(\rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left(\sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases}$$

Critical velocity

The flow monotonically flattens at infinity $\iff$ The stationary continuity equations are strongly elliptic:

$$\text{SE} \left[\rho = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{V}, \sigma = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{v} \right] > 0$$

$$\text{SE}(\mathbf{r}; V_0, \alpha, \beta, U, u) > 0$$

$$V_0 < V_c(\alpha, \beta, U, u)$$

Local generalization of the Landau criterion for an imbalanced mixture $w \gg 1$: LDA ✓

Chaplygin method

Sci. Mem. Moscow Univ. Math. Phys. **21**, 1 (1902)

Phys. Rev. Lett. **69**, 1644 (1992)

Phys. Rev. A **109**, 013317 (2024)

Superfluid hydrodynamics

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- Stationary continuity equations:

$$\text{Compressibility} \ll 1$$

$$\begin{cases} \nabla \cdot \left(\rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left(\sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases} \quad \begin{cases} \nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{v} = 0 \\ \text{Appropriate boundary conditions @ } r = w \end{cases}$$

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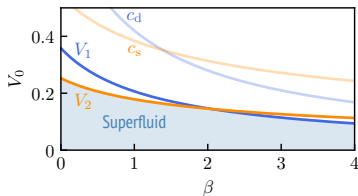
Phys. Rev. Lett. **69**, 1644 (1992)

Phys. Rev. A **109**, 013317 (2024)

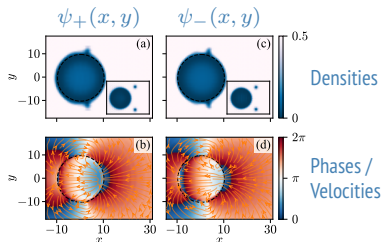
Impenetrable impurity

$$U > (1 + \alpha)/(1 + \beta)$$

$$V_c = \min \left\{ V_1: c_d \sqrt{\frac{(1 + \beta)[\sqrt{(1 + \beta)(121 + 25\beta)} - (11 + 5\beta)]}{9\beta}}, V_2: c_s \sqrt{\frac{2(1 + \alpha)}{11 + 5\alpha}} \right\}$$



Vortex-antivortex pairs at the impurity's poles

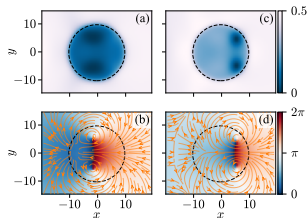


Penetrable impurity

Cumbersome yet analytical

$$V_c(\alpha, \beta, \boxed{U, u})$$

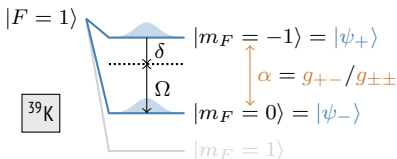
Jones-Roberts solitons inside the impurity



3. Nonlinear periodic waves in a binary BEC

In progress (2026)

BEC in two Rabi-coupled Zeeman states



$$\mathcal{H} = \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha|\psi_+|^2|\psi_-|^2 - \frac{\Omega}{2}(\psi_+^*\psi_- + \psi_-^*\psi_+) + \frac{\delta}{2}(|\psi_+|^2 - |\psi_-|^2)$$

Reminder: $\rho = |\psi_+|^2 + |\psi_-|^2$

Cubic-quintic nonlinear Schrödinger equation

Ground state for $\frac{|1-\alpha|\rho}{\Omega} \ll 1$:

D. Petrov, Orsay T. Bourdel, Palaiseau L. Tarruell, Barcelona

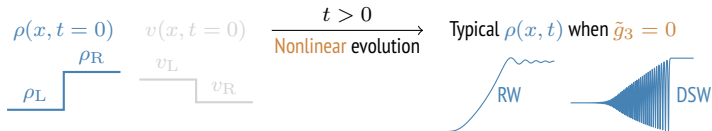
$$\mathcal{H}_{\text{gs}} \simeq -\frac{\sqrt{\Omega^2 + \delta^2}}{2}\rho + g_2\frac{\rho^2}{2} + \boxed{g_3\frac{\rho^3}{3}} \quad \begin{cases} g_2 = 1 - \frac{1}{2}\frac{1-\alpha}{1+\delta^2/\Omega^2} \\ g_3 = -\frac{3}{4}\frac{(1-\alpha)^2\delta^2/\Omega^2}{\Omega(1+\delta^2/\Omega^2)^{5/2}} \boxed{< 0} \end{cases}$$

$\Rightarrow$ Effective **cubic-quintic nonlinear** Schrödinger dynamics for a scalar field $\varphi(\mathbf{r}, t)$ with density $|\varphi|^2 = \rho$:

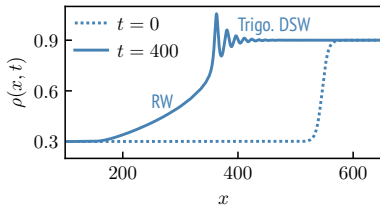
$$i\partial_t\varphi = \left(-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 + g_3|\varphi|^4\right)\varphi$$

Contact dispersive shock waves

- 1D configuration (BEC trapped in the $y-z$ plane): $i\partial_t \varphi = (-\frac{1}{2}\partial_{xx} + \tilde{g}_2|\varphi|^2 + \tilde{g}_3|\varphi|^4)\varphi$
- Riemann initial-value problem: $\varphi = \sqrt{\rho} e^{i \int dx v}$



- Contact dispersive shock waves emerge when $\tilde{g}_3 \neq 0$:



Whitham modulation theory

Nonlinear Periodic Waves and Their Modulations (2000)

Physica D **333**, 11 (2016)

Within experimental reach @ LCF:

$$(\Delta x, \Delta t)_{\text{Contact DSW}} \simeq (14.4 \mu\text{m}, 318.4 \text{ ms})$$

$$\begin{aligned} \omega_{\perp}/(2\pi) &= 300 \text{ Hz} & \Omega/(2\pi) &= 25.4 \text{ kHz} \\ \rho &\simeq 2.5 \times 10^9 \text{ m}^{-3} & \delta/\Omega &= 0.9 \end{aligned}$$

4. Vortex-induced self-propulsion against a superflow

arXiv:2512.09028 (2025) & Forthcoming (2026)



Myrann Baker-Rasooli



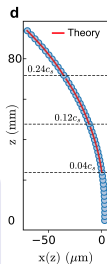
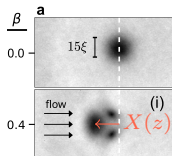
Tangui Aladjidi

@ LKB, Paris

Upstream motion driven by downstream vortices

$$i\partial_z\psi = [-\frac{1}{2}\nabla^2 + U(\mathbf{r} - X\hat{\mathbf{x}}) + |\psi|^2]\psi$$

$$k\frac{d^2X}{dz^2} \propto \int d^2\mathbf{r} |\psi|^2 \partial_x U(\mathbf{r} - X\hat{\mathbf{x}})$$

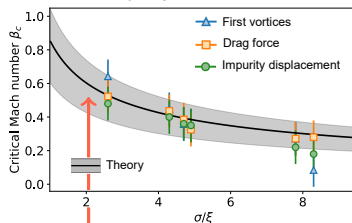


$$X(z) \simeq -C^{\text{st}} \Delta \rho z^2 + O(z)$$

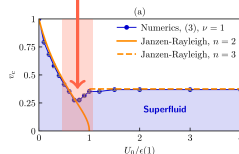
$$\Delta \rho \simeq \frac{2\sqrt{2}}{w} \left(V_0 - \frac{\sqrt{2}}{w} \right) \gtrsim 0.16$$

Critical speed for vortex shedding

V_c vs. impurity's radius w



$$V_c \simeq \sqrt{\frac{8}{11} \frac{\delta}{w}} \quad \delta \equiv w - r_{\text{v-av}} \sim 1$$



Phys. Rev. A
109, 013317 (2024)

See also:
Phys. Rev. A
107, 023310 (2023)

V_c vs. impurity's amplitude U_0 for $w \gg 1$

5. Summary and outlook

Critical speed of a binary superfluid of light

[arXiv:2601.16005 \(2026\)](#)

- $V_c = \min\{c_d, c_s\} \xleftarrow{0 \leftarrow U_{\pm}} V_c = \text{fn.}(U_{\pm})$
Landau criterion Strongly elliptic stationary flow
- $V_0 \gtrsim V_c$: Vortex-antivortex pairs & Jones-Roberts solitons in both components ψ_+ & ψ_-

Nonlinear periodic waves in a binary BEC

[In progress \(2026\)](#)

- Effective cubic-quintic nonlinear Schrödinger description at large Rabi frequency:
 $i\partial_t \varphi = (-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 - |g_3||\varphi|^4)\varphi$
- Contact dispersive shock waves (rarefaction + trigonometric) in a 1D configuration

Vortex-induced self-propulsion against a superflow

[arXiv:2512.09028 \(2025\)](#) & [Forthcoming \(2026\)](#)

- Upstream trajectory of the mobile impurity: $X(z) \simeq -C^{\text{st}}\Delta\rho z^2 + O(z)$
- Critical speed for vortex shedding: $V_c \simeq \sqrt{\frac{8}{11}}w^{-1/2}$

Ongoing work and future directions

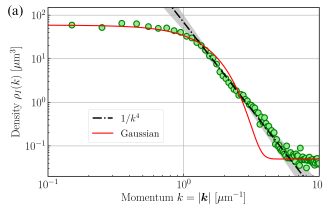
- Quantum depletion & OTOCs in a superfluid of light

Phys. Rev. A **92**, 043802 (2015)

$$[\hat{\psi}(\mathbf{r}, t; z), \hat{\psi}^\dagger(\mathbf{r}', t'; z)] = \frac{2\hbar\omega_0}{n_0\epsilon_0 c} \delta^{(2)}(\mathbf{r} - \mathbf{r}') \delta(t - t')$$

- Nonequilibrium Tan contact in a quenched binary BEC

Phys. Rev. Lett. **130**, 153401 (2023)



$$\rho(k, t) \underset{k \rightarrow \infty}{\simeq} \frac{\mathcal{C}(t)}{k^4}$$

- Quenched 2D soliton gas
- KPZ dynamics in quantum scars