

**Collective dynamics in binary superfluids:  
From dissipationless flow to dispersive shock waves**

Pierre-Élie Larré

LPTMS, Orsay



# 1. Introduction

## Binary superfluids...

- Condensates of interacting atoms in 2 hyperfine spin states · Proc. Internat. School Phys. Enrico Fermi **211** (2025)  
polaritons excitonic Rev. Mod. Phys. **85**, 299 (2013)
- Elliptically polarized laser in a birefringent nonlinear medium · ... Adv. At. Mol. Opt. Phys. **74**, 157 (2025)

$$\bullet \mathcal{H} = \frac{1}{2}(|\nabla\psi_+|^2 + |\nabla\psi_-|^2) + \underbrace{\frac{1}{2}(|\psi_+|^4 + |\psi_-|^4)}_{\text{Intra-component int.}} + \underbrace{\alpha|\psi_+|^2|\psi_-|^2}_{\text{Inter-component int.}} \quad \alpha = g_{+-}/g_{\pm\pm}$$

$$\bullet \psi_{\pm} = \sqrt{\rho_{\pm}} e^{i \int \mathbf{dr} \cdot \mathbf{v}_{\pm}}$$

$$\begin{array}{l} \text{Density mode} \\ \rho, \mathbf{V} \text{ fluct.} \end{array} \left\{ \begin{array}{l} \rho = \rho_+ + \rho_- \\ \mathbf{V} = \frac{1}{2}(\mathbf{v}_+ + \mathbf{v}_-) \end{array} \right. \quad \begin{array}{l} \text{Spin mode} \\ \sigma, \mathbf{v} \text{ fluct.} \end{array} \left\{ \begin{array}{l} \sigma = \rho_+ - \rho_- \\ \mathbf{v} = \mathbf{v}_+ - \mathbf{v}_- \end{array} \right.$$

- Miscibility problems · Collective & topological excitations · ...

## ...of light

Phys. Rev. Lett. **134**, 223403 (2025)

Elliptically polarized laser in a warm vapor of  $^{87}\text{Rb}$   
@ LKB, Paris



Quentin Glorieux



Claire Michel



Nicolas Cherroret

## ...of matter

Phys. Rev. Lett. **128**, 083401 (2022)

BEC of  $^{39}\text{K}$  in 2 coherently coupled Zeeman states  
@ LCF, Palaiseau



Thomas Bourdel

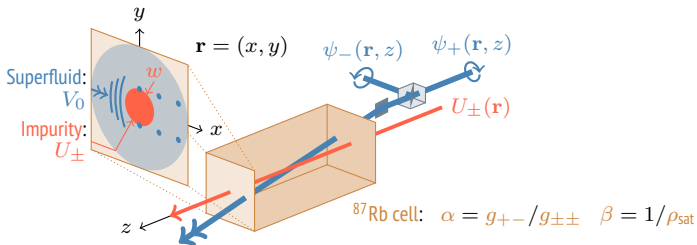


Thibault Congy



Patrick Sprenger

## 2. Critical speed of a binary superfluid of light



### Model equations

- 2-component inhomogeneous nonlinear Schrödinger-type equation, analogous to the Gross-Pitaevskii equation of Bose-Bose superfluid mixtures:

$$i\partial_z \psi_{\pm} = \left[ -\frac{1}{2}\nabla^2 + U_{\pm}(\mathbf{r}) + \frac{|\psi_{\pm}|^2 + \alpha|\psi_{\mp}|^2}{1 + \beta(|\psi_{+}|^2 + |\psi_{-}|^2)} \right] \psi_{\pm}$$

- What is the condition on the incident velocity  $V_0$  to ensure superfluid, dissipationless motion?

$$\psi_{\pm}^{(\text{in})} = \sqrt{\frac{1}{2}} e^{i(V_0 x - \mu z)}$$

Fully balanced

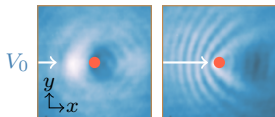
$$\frac{U_{\pm} > 0}{\text{Repulsive}}$$

$$\frac{g_{\pm\pm} > g_{+-} > 0}{\text{Miscible \& repulsive}} \implies 0 < \alpha < 1$$

## Linear response

- Impurity  $\ll$  Interactions
- Impurity-induced excitations:  
Density & spin Bogoliubov waves

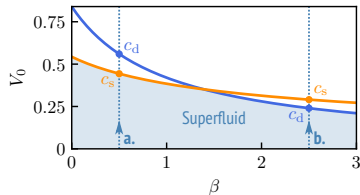
Single-component superfluid of light  
in a photorefractive crystal



Nat. Commun. 9, 2108 (2018)

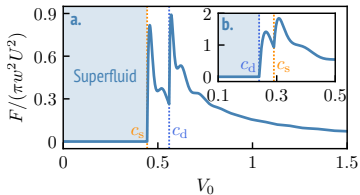
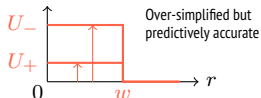
## Landau's criterion

$$V_0 < V_c = \min \left\{ \begin{aligned} c_d &= \sqrt{\frac{1+\alpha}{2}} \frac{1}{1+\beta}, \\ c_s &= \sqrt{\frac{1-\alpha}{2}} \frac{1}{\sqrt{1+\beta}} \end{aligned} \right\}$$



## Drag force

$$F = \int d^2\mathbf{r} \begin{bmatrix} \psi_+^* & \psi_-^* \end{bmatrix} \begin{bmatrix} \partial_x U_+(\mathbf{r}) & 0 \\ 0 & \partial_x U_-(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$$



## Superfluid hydrodynamics

- Stationary Bernoulli equations:

$$\cos \theta = \sigma / \rho \quad U, u = U_+ \pm U_-$$

$$\begin{cases} \frac{(1 + \alpha)\rho}{1 + \beta\rho} = \frac{1 + \alpha}{1 + \beta} - U \mathbf{1}_{r < w} - (V^2 - V_0^2) - \frac{v^2}{4} + \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \tan \theta} - \frac{(\nabla \theta)^2}{4} \\ \frac{(1 - \alpha)\sigma}{1 + \beta\rho} = -u \mathbf{1}_{r < w} - \mathbf{V} \cdot \mathbf{v} - \frac{\nabla \cdot (\rho \nabla \theta)}{2\rho \sin \theta} \end{cases}$$

- Stationary continuity equations:

$$\begin{cases} \nabla \cdot \left( \rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left( \sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases}$$

## Elliptic flow

The flow **monotonically** flattens at infinity  $\iff$  The stationary continuity equations are **strongly elliptic**

### Chaplygin's method

Sci. Mem. Moscow Univ. Math. Phys. 21, 1 (1902)

Phys. Rev. Lett. 69, 1644 (1992)

Phys. Rev. A 109, 013317 (2024)

$$\text{StrEll} \left[ \rho = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{V}, \sigma = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{v} \right] > 0$$

$$\text{StrEll}(\mathbf{r}; V_0, \alpha, \beta, U, u) > 0, \quad \forall \mathbf{r} = (x, y)$$

**Local-field** generalization of Landau's criterion for a **fully imbalanced** mixture  $w \gg 1$ : LDA  $\checkmark$

$$\iff V_0 < V_c(\alpha, \beta, U, u)$$

## Superfluid hydrodynamics

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## Elliptic flow

The flow monotonically flattens at infinity  $\iff$  The stationary continuity equations are strongly elliptic

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$$\text{StrEll} \left[ \rho = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{V}, \sigma = \frac{\text{fn.}(\mathbf{V}, \mathbf{v})}{\text{Bernoulli}}, \mathbf{v} \right] > 0$$

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- Stationary continuity equations:

Compressibility  $\ll 1$

$$\begin{cases} \nabla \cdot \left( \rho \mathbf{V} + \frac{\sigma \mathbf{v}}{2} \right) = 0 \\ \nabla \cdot \left( \sigma \mathbf{V} + \frac{\rho \mathbf{v}}{2} \right) = 0 \end{cases} \quad \begin{cases} \nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{v} = 0 \\ \text{Appropriate boundary conditions @ } r = w \end{cases}$$

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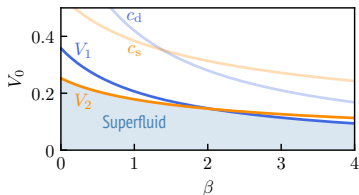
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$$\iff V_0 < V_c(\alpha, \beta, U, \mathbf{u})$$

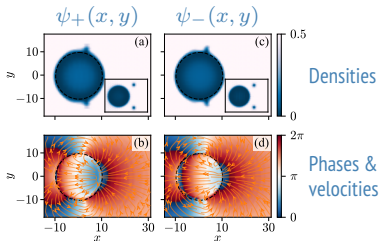
## Impenetrable impurity

$$U > (1 + \alpha)/(1 + \beta)$$

$$V_c = \min \left\{ V_1: c_d \sqrt{\frac{(1 + \beta)[\sqrt{(1 + \beta)(121 + 25\beta)} - (11 + 5\beta)]}{9\beta}}, V_2: c_s \sqrt{\frac{2(1 + \alpha)}{11 + 5\alpha}} \right\}$$



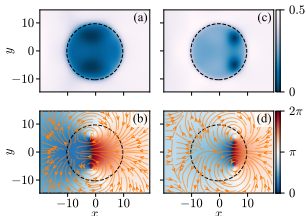
Vortex-antivortex pairs at the impurity's poles



## Penetrable impurity

Cumbersome yet analytical  $V_c(\alpha, \beta, U, u)$

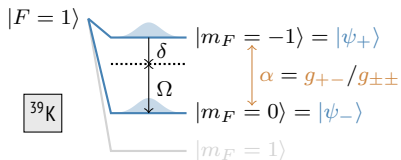
Jones-Roberts solitons inside the impurity



### 3. Nonlinear periodic waves in a binary BEC

Forthcoming (2026)

#### BEC in two Rabi-coupled Zeeman states



$$\mathcal{H} = \frac{1}{2}(|\psi_+|^4 + |\psi_-|^4) + \alpha|\psi_+|^2|\psi_-|^2 - \frac{\Omega}{2}(\psi_+^*\psi_- + \psi_-^*\psi_+) + \frac{\delta}{2}(|\psi_+|^2 - |\psi_-|^2)$$

Reminder:  $\rho = |\psi_+|^2 + |\psi_-|^2$

#### Cubic-quintic nonlinear Schrödinger equation

Ground state for  $\frac{|1-\alpha|\rho}{\Omega} \ll 1$ :

D. Petrov, Orsay T. Bourdel, Palaiseau L. Tarruell, Barcelona

$$\mathcal{H}_{\text{gs}} \simeq -\frac{\sqrt{\Omega^2 + \delta^2}}{2}\rho + g_2\frac{\rho^2}{2} + \boxed{g_3\frac{\rho^3}{3}}$$

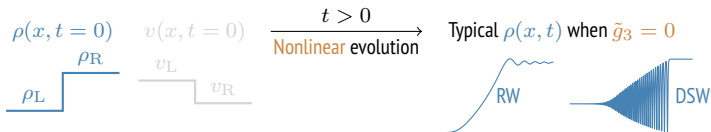
$$\begin{cases} g_2 = 1 - \frac{1}{2}\frac{1-\alpha}{1+\delta^2/\Omega^2} \\ g_3 = -\frac{3}{4}\frac{(1-\alpha)^2\delta^2/\Omega^2}{\Omega(1+\delta^2/\Omega^2)^{5/2}} \boxed{< 0} \end{cases}$$

$\Rightarrow$  Effective **cubic-quintic nonlinear** Schrödinger dynamics for a scalar field  $\varphi(\mathbf{r}, t)$  with density  $|\varphi|^2 = \rho$ :

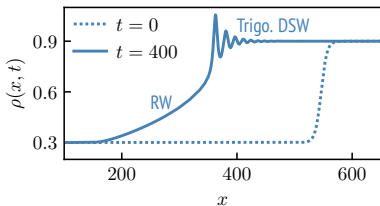
$$i\partial_t\varphi = \left(-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 + g_3|\varphi|^4\right)\varphi$$

## Contact dispersive shock waves

- 1D configuration (BEC trapped in the  $y-z$  plane):  $i\partial_t\varphi = (-\frac{1}{2}\partial_{xx} + \tilde{g}_2|\varphi|^2 + \tilde{g}_3|\varphi|^4)\varphi$
- Riemann initial-value problem:  $\varphi = \sqrt{\rho} e^{i \int dx v}$



- Contact DSWs emerge when  $\tilde{g}_3 \neq 0$ :



### Whitham modulation theory

*Nonlinear Periodic Waves and Their Modulations* (2000)  
 Physica D 333, 11 (2016)

Within experimental reach @ LCF:

$$(\Delta x, \Delta t)_{\text{Contact DSW}} \simeq (14.4 \mu\text{m}, 318.4 \text{ms})$$

$$\begin{aligned} \omega_{\perp}/(2\pi) &= 300 \text{ Hz} & \Omega/(2\pi) &= 25.4 \text{ kHz} \\ \rho &\simeq 2.5 \times 10^9 \text{ m}^{-1} & \delta/\Omega &= 0.9 \end{aligned}$$

# 4. Vortex-induced propulsion against a superflow

Phys. Rev. Lett. **136**, 223401 (2026) & Forthcoming (2026)



Myrann Baker-Rasooli

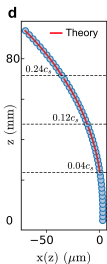
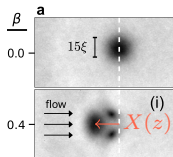
Tangui Aladjidi

@ LKB, Paris

## Upstream motion driven by downstream vortices

$$i\partial_z\psi = \left[-\frac{1}{2}\nabla^2 + U(\mathbf{r} - X\hat{x}) + |\psi|^2\right]\psi$$

$$\frac{d^2X}{dz^2} \propto \int d^2\mathbf{r} |\psi|^2 \partial_x U(\mathbf{r} - X\hat{x})$$

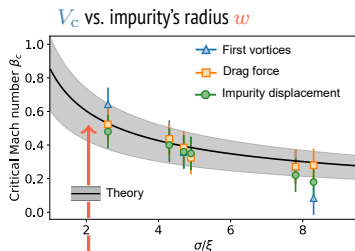


$$X(z) \simeq -C^{\text{st}} \Delta\rho z^2 + O(z)$$

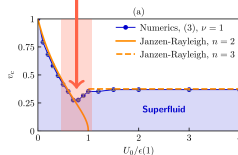
$$\Delta\rho \simeq \frac{2\sqrt{2}}{w} \left( V_0 - \frac{\sqrt{2}}{w} \right)$$

$\gtrsim 0.16$

## Critical speed for vortex shedding



$$V_c \simeq \sqrt{\frac{8}{11} \frac{\delta}{w}} \quad \delta = w - r_{\text{v-av}} \sim 1$$



Phys. Rev. A  
**109**, 013317 (2024)

See also:  
Phys. Rev. A  
**107**, 023310 (2023)

$V_c$  vs. impurity's amplitude  $U_0$  for  $w \gg 1$

## 5. Summary and outlook

### Critical speed of a binary superfluid of light

Eur. Phys. J. D **80**, 72 (2026) *Topical Collection*  
"Paraxial Fluids of Light"

- $V_c = \min\{c_d, c_s\}$  ←  $U_{\pm}$  Landau's criterion  $V_c = \text{fn.}(U_{\pm})$  Strongly elliptic stationary flow
- $V_0 \gtrsim V_c$ : Vortex-antivortex pairs & Jones-Roberts solitons in both components  $\psi_+$  &  $\psi_-$

### Nonlinear periodic waves in a binary BEC

Forthcoming (2026)

- Effective **cubic-quintic nonlinear** Schrödinger description at large Rabi frequency:  
 $i\partial_t \varphi = (-\frac{1}{2}\nabla^2 + g_2|\varphi|^2 + g_3|\varphi|^4)\varphi$
- Contact DSWs (RW glued to trigo. DSW) in a 1D configuration

### Vortex-induced propulsion against a superflow

Phys. Rev. Lett. **136**, 223401 (2026) & Forthcoming (2026)

- Upstream trajectory of the mobile impurity:  $X(z) \simeq -c^{\text{st}}\Delta\rho z^2 + O(z)$
- Critical speed for vortex shedding:  $V_c \simeq \sqrt{\frac{8}{11}}w^{-1/2}$

## Ongoing work and future directions

- **Quantum** depletion of a **3D** superfluid of light × LKB Phys. Rev. A **92**, 043802 (2015)
- **KPZ** dynamics in a tower of **quantum many-body scars** × LPTMS Phys. Rev. B **112**, 134314 (2025)
- Post-**quench** dynamics of a **disordered** superfluid of light × INPHYNI EPL **132**, 66001 (2021)
- Post-**quench** dynamics of a quantum **mixture** × LCF Phys. Rev. Lett. **130**, 153401 (2023)
- **Rayleigh-Plesset** dynamics in an **immiscible** quantum mixture × LPTMS

Merci !