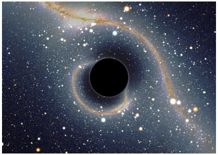
Quantum fluctuations and nonlinear effects in Bose–Einstein condensates: From dispersive shock waves to acoustic Hawking radiation

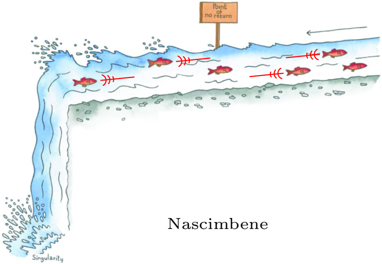
Pierre-Élie Larré

Laboratoire de Physique Théorique et Modèles Statistiques
Université Paris 11 — Orsay

September 20, 2013



Riazuelo
Institut d'Astrophysique de Paris



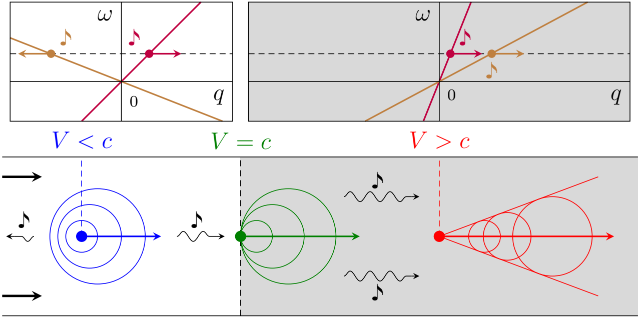
Nascimbene

$\boxed{\mathbf{1}}$

Acoustic black holes in Bose–Einstein condensates

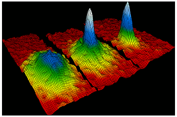
One-dimensional acoustic black holes

Phonons: $\omega - Vq = \pm c\, q$



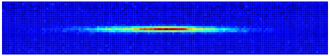
Exterior of the sonic hole

Interior of the sonic hole



Anderson *et al.*, *Science* (1995)

Quasi-1D atomic condensate



Institut d'Optique

Acoustic black holes in quasi-one-dimensional atomic condensates

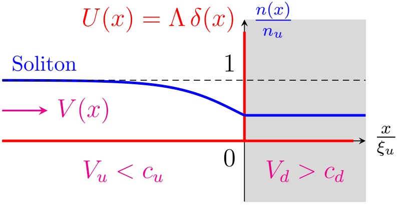
Stationary Gross–Pitaevskii equation

$$i\hbar\,\partial_t\hat{\Psi} = \left[-\frac{\hbar^2}{2m}\partial_{xx} + U(x) + g\,\hat{n} - \mu\right]\hat{\Psi}$$

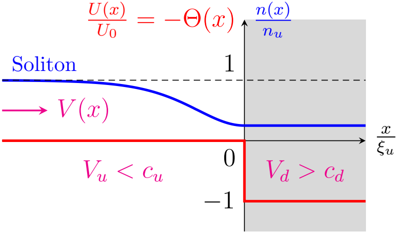
- $\hat{\Psi}(x,t)$: Heisenberg field operator
- $\hat{n}(x,t) = \hat{\Psi}^\dagger(x,t)\,\hat{\Psi}(x,t)$: density operator
- $U(x)$: external potential
- $g > 0$: four-field coupling constant
- μ: chemical potential

$$\boxed{\mu\,\Psi = \left[-\frac{\hbar^2}{2m}\partial_{xx} + U(x) + g\,n\right]\Psi}$$

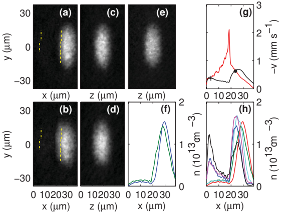
- $\Psi(x) = \langle\hat{\Psi}(x,t)\rangle$: order parameter
- $n(x) = |\Psi(x)|^2$: longitudinal density



δ-peak configuration



Waterfall configuration



Lahav *et al.* *PRL* (2010)

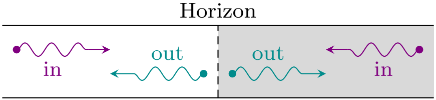
Larré, Recati, Carusotto and Pavloff
PRA **85**, 013621 (2012)

Quantum fluctuations around the background: Bogoliubov approach

$$\boxed{\hat{\Psi}(x,t) = \Psi(x) + \hat{\psi}(x,t) \quad \text{with} \quad \hat{\psi} \ll \Psi}$$

Bogoliubov spectrum

$$\omega - V\,q = \pm\, c\, q \sqrt{1 + \frac{\xi^2 q^2}{4}}$$

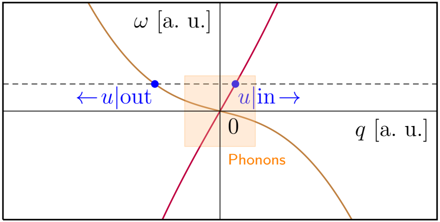


Scattering matrix

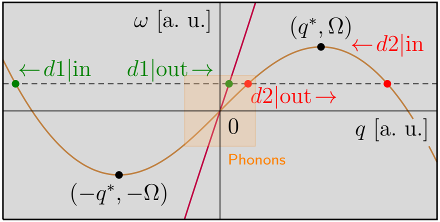
$$\begin{bmatrix} u|\text{out} \\ d1|\text{out} \\ (d2|\text{out})^\dagger \end{bmatrix} = S(\omega) \begin{bmatrix} u|\text{in} \\ d1|\text{in} \\ (d2|\text{in})^\dagger \end{bmatrix}$$

$|S_{i,j}(\omega)|^2$: transmission/reflection coefficient for a j-ingoing mode oscillating at pulsation ω scatters into a i-outgoing mode

Subsonic flow



Supersonic flow



One-body Hawking signal

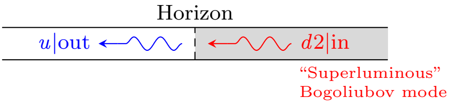
Radiated power

• Energy current associated to the emission of elementary excitations:

$$\hat{\Pi}(x,t) = -\frac{\hbar^2}{2m}\,\partial_t\hat{\Psi}^\dagger(x,t)\,\partial_x\hat{\Psi}(x,t) + \text{H.c.}$$

• Deep outside the black hole and at zero temperature:

$$\left\langle \hat{\Pi}(-\infty,t)\right\rangle_{T=0} = -\int_0^{\Omega}\frac{\mathrm{d}\omega}{2\pi}\,\hbar\,\omega\,|S_{u,d2}(\omega)|^2$$



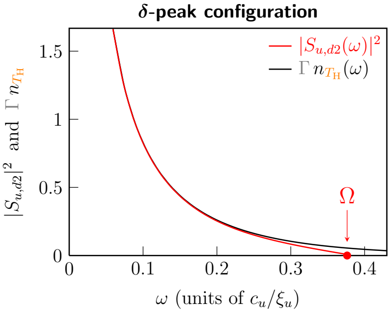
Radiation spectrum

$$|S_{u,d2}(\omega)|^2 \simeq \Gamma\, n_{T_{\mathrm{H}}}(\omega) = \frac{\Gamma}{\exp\left(\frac{\hbar\,\omega}{T_{\mathrm{H}}}\right)-1}$$

Hawking temperature

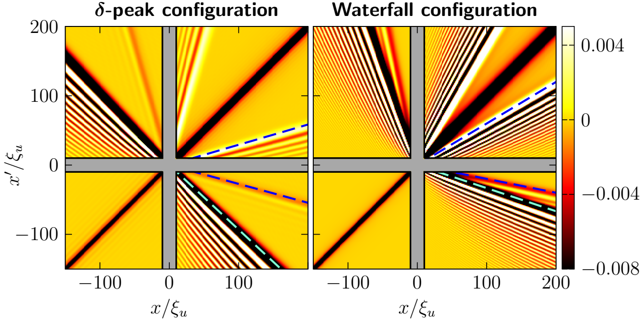
Knowledge of the exact low-ω behavior of $S_{u,d2}(\omega)$ up to $\mathcal{O}(\sqrt{\omega}) \Longrightarrow$ Analytical estimates of the gray-body factor and the Hawking temperature:

$$\boxed{T_{\mathrm{H}} \sim 10\,\mathrm{nK} \;<\; T_{\mathrm{exp}} \sim 100\,\mathrm{nK}}$$



Two-body Hawking signal

$$\boxed{\begin{array}{c} g^{(2)}(x, x') = \\ \langle : \delta\hat{n}(x, t)\, \delta\hat{n}(x', t) : \rangle \end{array}}$$

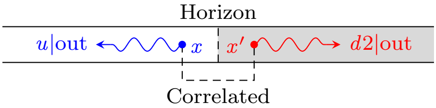


At time t after their emission the Hawking phonons $u|\text{out}$ and $d2|\text{out}$ are respectively located at

$$x = V_{\text{g}}(q_{u|\text{out}})t \quad \text{and} \quad x' = V_{\text{g}}(q_{d2|\text{out}})t,$$

inducing a correlation signal in the $\{x, x'\}$ plane along the line of slope

$$x'/x = V_{\text{g}}(q_{d2|\text{out}})/V_{\text{g}}(q_{u|\text{out}}).$$

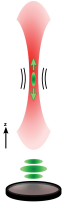


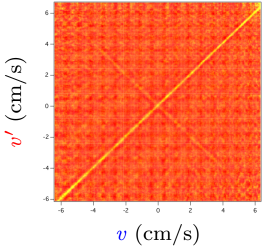
Compressibility sum rule

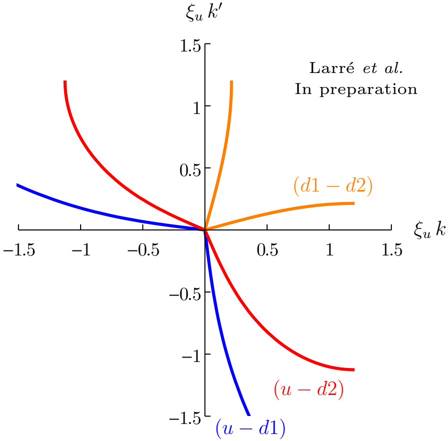
$$\int_{\mathbb{R}} \mathrm{d}x'\, g^{(2)}(x, x') = -n(x) \qquad \begin{pmatrix} T = 0 \\ |x| \to \infty \end{pmatrix}$$

Two-body Hawking signal in momentum space

Jaskula *et al.*, *PRL* (2012)







$$\hat{N}(k,t) = \hat{\Psi}^\dagger(k,t)\,\hat{\Psi}(k,t)$$

$$\boxed{\begin{array}{c} g^{(2)}(k,k') = \\ \langle \hat{N}(k,t)\,\hat{N}(k',t)\rangle - \langle \hat{N}(k,t)\rangle\langle \hat{N}(k',t)\rangle \end{array}}$$

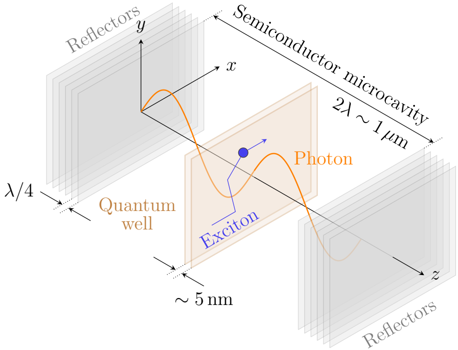
1 Acoustic black holes in Bose–Einstein condensates: Conclusions

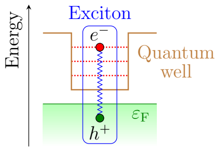
- Bose–Einstein condensates offer interesting prospects to observe a spontaneous—so fully quantum—Hawking-like radiation.
- New sonic-hole configurations of experimental interest
- Analytical formula for the Hawking temperature $T_{\rm H}$; $T_{\rm H} \ll T_{\rm exp}$: the one-body Hawking signal is lost in the thermal noise, but...
- ... nonlocal two-body correlations (in position and momentum space) provide a clear qualitative signature of the occurrence of Hawking radiation, even at finite temperature.
- The compressibility sum rule at zero temperature is verified in the presence of an acoustic horizon.

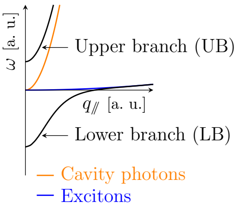
$\boxed{2}$

Waves in the flow of a polariton condensate

Microcavity polaritons



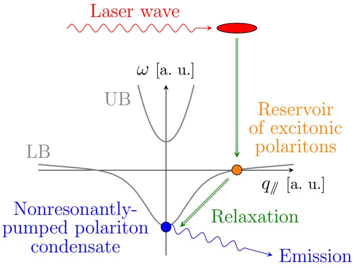


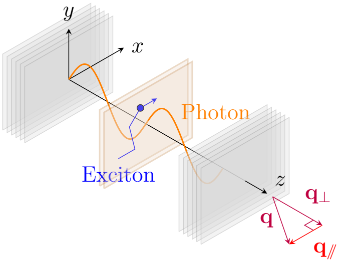


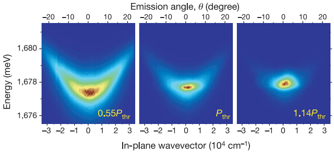
- $\frac{1}{\sqrt{2}}\Big(|\text{Photon}\rangle + |\text{Exciton}\rangle\Big) = |\text{Polariton}\rangle$
- Photon, exciton: bosons $\Longrightarrow$ Polariton: boson
- Polariton effective mass (LB): $m \lesssim 10^{-4}\, m_{\mathrm{e}}$
- Polariton lifetime: $\tau \lesssim 50\,\mathrm{ps}$

Polariton condensation

- Interacting bosons
- Spontaneous appearance of temporal coherence and long-range spatial coherence
- Low mass $\Longrightarrow$ High $T_c \sim 10\,\mathrm{K}$
- Finite polariton-lifetime $\Longrightarrow$ "Direct" access to the internal properties of the polariton fluid by detection of the light emitted by the gas: no intrusive measurements







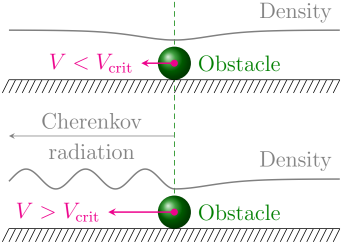
Kasprzak *et al.*, *Nature* (2006)

- Grenoble: Institut Néel
- Lausanne: EPFL

Superfluidity in polariton condensates

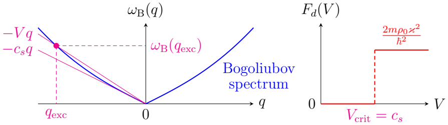
Landau criterion

- Weakly perturbing obstacle moving at constant velocity V in a *conservative* quantum fluid at *zero temperature*
- $\Longrightarrow$ There can exist a critical velocity V_{crit} such that:
  - When $V < V_{\text{crit}}$, no excitation is emitted away from the obstacle and there is no drag force: $F_d = 0$ (superfluid regime);
  - When $V > V_{\text{crit}}$, a Cherenkov radiation of linear waves occurs and the obstacle is subject to a finite drag-force: $F_d \neq 0$ (dissipative regime).

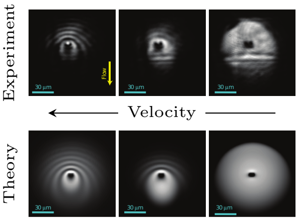


Polariton condensates: *nonconservative* quantum fluids, but

$U_{\text{ext}}(x,t) = \varkappa\,\delta(x + Vt)$ in a 1D atomic BEC



Amo *et al.*, *Nat. Phys.* (2009)



Nonresonantly-pumped polariton condensates at zero temperature: a simple one-dimensional model

Phenomenological modification of the Gross–Pitaevskii equation

$$\boxed{\mathrm{i}\partial_t\psi = -\tfrac{1}{2}\partial_{xx}\psi + U_{\mathrm{ext}}(x,t)\,\psi + \rho\,\psi + \underline{\mathrm{i}\eta\,(1-\rho)\,\psi}}$$

- $\psi(x,t)$: condensate wavefunction (scalar because $\cancel{\sigma = \pm 1}$)
- $\rho(x,t) = |\psi(x,t)|^2$: longitudinal density
- $U_{\mathrm{ext}}(x,t)$: potential of an external obstacle

$\partial_t\psi = \eta\,\psi$	$\eta =$ (Gains due to pumping) − (Losses $\propto 1/\tau$) > 0
$\partial_t\psi = -\eta\,\lvert\psi\rvert^2\,\psi$	Gain saturation
$\Longrightarrow \partial_t\psi\rvert_{\mathrm{tot}} = \eta\,(1-\lvert\psi\rvert^2)\,\psi$	Dynamical equilibrium between gains and losses
	$\Longrightarrow$ Steady-state configuration with $\lvert\psi_0\rvert^2 = 1 < \infty$

Uniform and stationary solution in the absence of external obstacle:

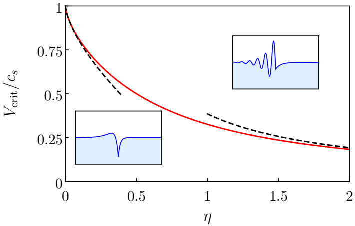
$$\psi_0(x,t) = \mathrm{e}^{-\mathrm{i}t}$$

$$\rho_0(x,t) = |\psi_0(x,t)|^2 = 1$$

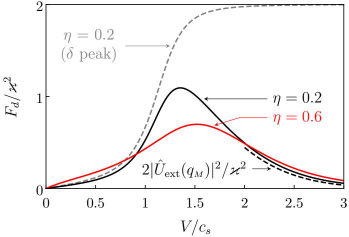
Finite-size obstacle moving at constant velocity $-V\hat{\mathbf{x}}$, $V > 0$:

$$U_{\mathrm{ext}}(X = x + Vt) \xrightarrow[|X|\to\infty]{} 0$$

Flow past a weakly perturbing impurity: From viscous drag to wave resistance



$$U_{\mathrm{ext}}(X) = \frac{\varkappa}{\sigma\sqrt{\pi}}\,\mathrm{e}^{-X^2/\sigma^2}$$



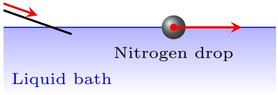
Larré, Kamchatnov and Pavloff
PRB **86**, 165304 (2012)

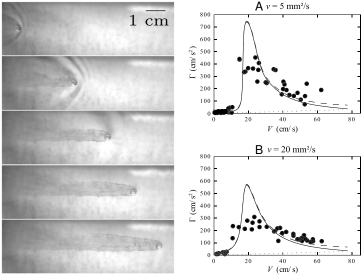
Counterintuitive effect

$\boxed{\max F_d \searrow \quad \text{when} \quad \eta \nearrow}$

Viscous effects reduce the amplitude of the wake and diminish the wave resistance which is the dominant source of drag when $V > V_{\mathrm{crit}}$.

Le Merrer *et al.*
PNAS (2011)





2 Waves in the flow of a polariton condensate: Conclusions

• Analysis of the one-dimensional flow of a nonresonantly-pumped scalar polariton condensate past a localized obstacle at zero temperature

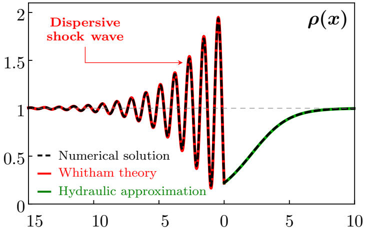
---

• **Weak-perturbation limit**: smooth crossover from a viscous flow to a regime where the drag is mainly dominated by wave resistance

• Onset of (damped) Cherenkov radiation at a velocity $V_{\mathrm{crit}}(\eta)/c_s \leqslant 1$ only depending on the damping parameter η ($\sim$ pumping and losses processes in the system)

• Absence of long-range wake $\neq$ absence of dissipation

• Whitham modulation theory and hydraulic approximation in the case of a supersonic fluid flowing past a **δ-peak impurity of arbitrary amplitude**



$\boxed{3}$

Hawking radiation in a two-component condensate

Polarization hydrodynamics in a spinor polariton condensate

Phenomenological model

$$
\begin{aligned}
\mathrm{i}\hbar\,\partial_t\psi_\sigma = &-\frac{\hbar^2}{2m}\partial_{xx}\psi_\sigma \\
&+ U_{\text{ext}}(x,t)\,\psi_\sigma - \sigma\,\hbar\,\Omega\,\psi_\sigma \\
&+ \left(g_1|\psi_\sigma|^2 + g_2|\psi_{-\sigma}|^2\right)\psi_\sigma \\
&+ \mathrm{i}\,(\gamma - \Gamma\rho)\,\psi_\sigma
\end{aligned}
$$

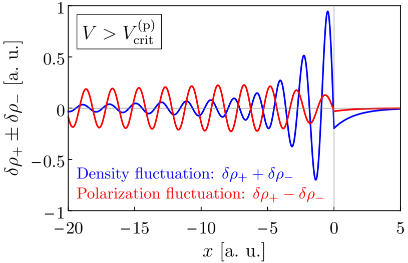
- $\sigma = \pm 1$: spin projections onto the z axis
- $\psi_\pm(x,t)$: condensate wavefunction
- $\rho(x,t) = |\psi_+|^2 + |\psi_-|^2$: longitudinal density
- $U_{\text{ext}}(x,t)$: potential of a finite-size obstacle moving at constant velocity $-V\hat{\mathbf{x}}$
- $2\,\hbar\,\Omega \propto B_z$: Zeeman splitting between the polarized states ψ_+ and ψ_-
- g_1, g_2: interactions between polaritons with parallel (g_1) and antiparallel (g_2) spins; repulsion dominates: typically,

$$-g_1/10 \sim g_2 < 0 < g_1$$

Linearized theory

- $V > V_{\text{crit}}^{(\text{d})}$: Cherenkov radiation of *damped* density-waves
- $V > V_{\text{crit}}^{(\text{p})} > V_{\text{crit}}^{(\text{d})}$: Cherenkov radiation of *weakly damped* polarization-waves

$U_{\text{ext}}(x,t) = \varkappa\,\delta(x + Vt)$



Larré, Kamchatnov and Pavloff
arXiv:1309.3494 (2013)

Kamchatnov, Kartashov, Larré and Pavloff
arXiv:1308.0784 (2013)

A simple black-hole configuration in the one-dimensional flow of a two-component condensate

- $\mathrm{i}\hbar\,\partial_t\hat{\Psi}_\sigma = \left[-\frac{\hbar^2}{2m}\partial_{xx} + U(x) + g_1\,\hat{\Psi}^\dagger_\sigma\hat{\Psi}_\sigma + g_2(x)\,\hat{\Psi}^\dagger_{-\sigma}\hat{\Psi}_{-\sigma} - \mu\right]\hat{\Psi}_\sigma \qquad 0 < g_2(x) < g_1$

- Step-like configuration:

  $U(x) = U_u\,\Theta(-x) + U_d\,\Theta(x)$

  $g_2(x) = g_{2,u}\,\Theta(-x) + g_{2,d}\,\Theta(x)$

  $\mu(\cancel{x}) = \frac{\hbar^2 k_0^2}{2m} + U(x) + \frac{1}{2}[g_1 + g_2(x)]n_0$

  Homogeneous and stationary flow:

  $\langle\hat{\Psi}_\pm(x,t)\rangle = \sqrt{\frac{n_0}{2}}\,\mathrm{e}^{\mathrm{i}k_0 x}, \quad \hbar\,k_0 = m\,V_0$

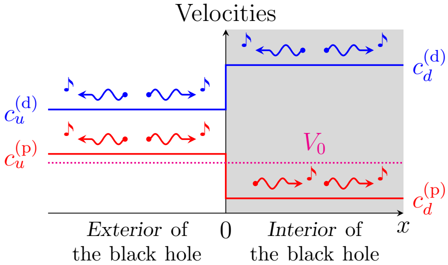
Acoustic horizon for the polarization phonons

- Long-wavelength elementary excitations:

  Polarization: $c^{(\mathrm{p})}(x) = \sqrt{\frac{n_0}{2m}[g_1 - g_2(x)]}$

  Density: $c^{(\mathrm{d})}(x) = \sqrt{\frac{n_0}{2m}[g_1 + g_2(x)]}$

- $c_d^{(\mathrm{p})} < V_0 < c_u^{(\mathrm{p})} < c_u^{(\mathrm{d})} < c_d^{(\mathrm{d})}$

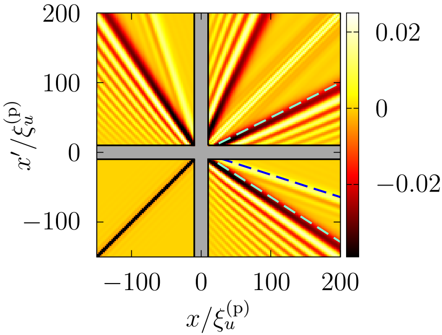


Larré and Pavloff, arXiv:1307.2843 (2013)
To appear in *EPL*

Hawking radiation of polarization waves

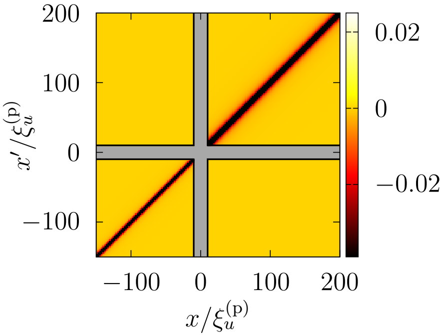
$$\hat{\pi}(x,t) = \hat{\Psi}_+^\dagger \hat{\Psi}_+ - \hat{\Psi}_-^\dagger \hat{\Psi}_-$$

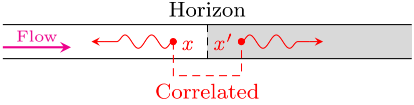
$$\boxed{g^{(\mathrm{p})}(x,x') = \langle : \hat{\pi}(x,t)\, \hat{\pi}(x',t) : \rangle}$$



$$n_0 + \delta\hat{n}(x,t) = \hat{\Psi}_+^\dagger \hat{\Psi}_+ + \hat{\Psi}_-^\dagger \hat{\Psi}_-$$

$$\boxed{g^{(\mathrm{d})}(x,x') = \langle : \delta\hat{n}(x,t)\, \delta\hat{n}(x',t) : \rangle}$$





Configuration where $\cancel{U(x)}$ and $\cancel{g_2(x)}$:

$$\frac{\xi^{(\mathrm{p},\mathrm{d})}}{n_0}\, g^{(\mathrm{p},\mathrm{d})}(x,x') = -\frac{1}{\pi}\, \frac{\xi^{(\mathrm{p},\mathrm{d})}}{|x-x'|} \int_0^\infty \frac{\mathrm{d}t}{(1+t^2)^{3/2}} \sin\left(2\, \frac{|x-x'|}{\xi^{(\mathrm{p},\mathrm{d})}}\, t\right)$$

3 Hawking radiation in a two-component condensate: Conclusions

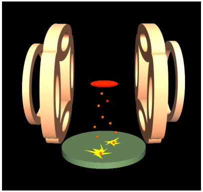
• Analysis of the one-dimensional flow of a nonresonantly-pumped spinor polariton condensate past a small localized obstacle at zero temperature and in the presence of a magnetic field transverse to the condensate

• Ejection of a weakly damped polarization-wave: does it make it possible to probe Hawking-like radiation in spinor polariton condensate?

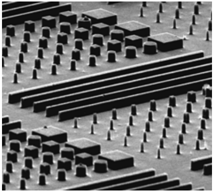
• Simple realization of an acoustic horizon in the flow of a one-dimensional two-component condensate

• The horizon affects only the polarization modes and not the density ones.

• The (one- and the) two-body signal associated to the analog of spontaneous Hawking radiation consists only in the emission of polarization waves.



Palaiseau: LCF



Marcoussis: LPN