Collective phenomena in two-component superfluids of light and matter: From dissipationless flow to dispersive shock waves

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▷ Quantum-SOPHA
▷ STLight
▷ UniQ-RingS

Two-component superfluids…

- Ultracold gases · Polariton condensates · Nonlinear lasers…
- $\psi_\pm(\mathbf{r},t) = \sqrt{\rho_\pm(\mathbf{r},t)}\,\exp[\mathrm{i}\int^{\mathbf{r}} \mathrm{d}\mathbf{r}' \cdot \mathbf{v}_\pm(\mathbf{r}',t)]$

Density mode $\delta\rho, \delta\mathbf{V}$ $\begin{cases} \rho = \rho_+ + \rho_- \\ \mathbf{V} = (\mathbf{v}_+ + \mathbf{v}_-)/2 \end{cases}$ Spin mode $\delta\sigma, \delta\mathbf{v}$ $\begin{cases} \sigma = \rho_+ - \rho_- \\ \mathbf{v} = \mathbf{v}_+ - \mathbf{v}_- \end{cases}$

- Feshbach resonances · Impurity problem · Dissipationless flow · Superfluid ferromagnetic phases · Collective excitations, e.g., magnetic solitons & vortices · Short- & long-range interactions within & beyond the mean-field regime…

…of light

Elliptically polarized laser in a birefringent nonlinear medium @ LKB, Paris

Quentin Glorieux
Clara Piekarski

Nicolas Cherroret

…of matter

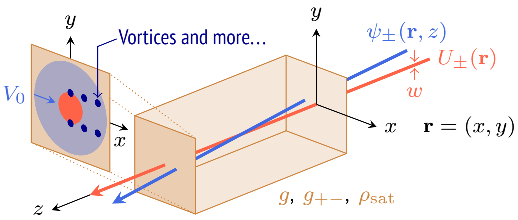
Coherently driven two-component Bose-Einstein condensate @ LCF, Palaiseau

Thomas Bourdel

Thibault Congy

Patrick Sprenger

Dissipationless flow of a two-component superfluid of light past a polarized obstacle



- Two-component inhomogeneous nonlinear Schrödinger-type equation:

$$\mathrm{i}\partial_z \psi_\pm = \left[-\frac{1}{2k}\nabla^2 + U_\pm(\mathbf{r}) + \frac{g|\psi_\pm|^2 + g_{+-}|\psi_\mp|^2}{1 + (|\psi_+|^2 + |\psi_-|^2)/\rho_{\rm sat}} \right] \psi_\pm$$

Analogous to the Gross-Pitaevskii equation of Bose-Bose superfluid mixtures
$\Longrightarrow$ Two-component "superfluid of light" evolving in time z in the $x\!-\!y$ plane

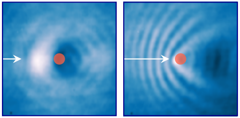
- Conditions for the following flow past the polarized obstacle to be dissipationless?

$$\psi_\pm \underset{\text{Infinity}}{\simeq} \sqrt{\frac{\rho_0}{2}}\, \mathrm{e}^{\mathrm{i}(kV_0 x - \mu z)} \qquad g > g_{+-} > 0$$

Also worth studying: Unbalanced $\rho_{\pm,0}$ & $\mathbf{v}_{\pm,0}$

Linear response

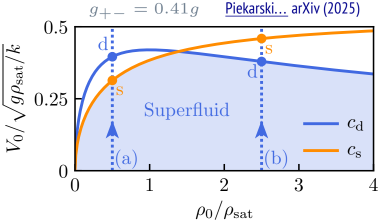
- Obstacle ≪ Interactions
- Obstacle-induced excitations: Density & spin
  Bogoliubov-Cherenkov waves



Michel… Nat. Commun. (2018)

Landau criterion

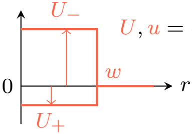
$V_0 < c_\mathrm{d}, c_\mathrm{s}$ = Bogoliubov speeds of sound



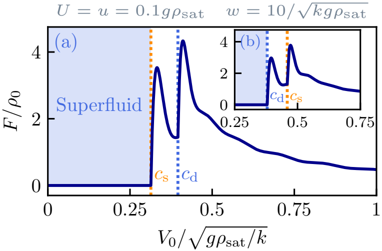
Drag force

$$F = \int_{\mathbf{r}} [\psi_+^* \ \psi_-^*] \begin{bmatrix} \partial_x U_+(\mathbf{r}) & 0 \\ 0 & \partial_x U_-(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$$

Over-simplified but predictively accurate



$U, u = U_+ \pm U_-$



Superfluid hydrodynamics $\cos\theta = \sigma/\rho$

Momentum

$$\begin{cases} \dfrac{(g+g_{+-})\rho}{1+\rho/\rho_{\rm sat}} = 2\mu(\rho_0, V_0) - U(\mathbf{r}) - kV^2 - \dfrac{kv^2}{4} \\ \qquad + \dfrac{\nabla^2\sqrt{\rho}}{k\sqrt{\rho}} + \dfrac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\tan\theta} - \dfrac{(\nabla\theta)^2}{4k} \\ \dfrac{(g-g_{+-})\sigma}{1+\rho/\rho_{\rm sat}} = -u(\mathbf{r}) - k\mathbf{V}\cdot\mathbf{v} - \dfrac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\sin\theta} \end{cases}$$

Mass

$$\begin{cases} \nabla\cdot\left(\rho\mathbf{V} + \dfrac{\sigma\mathbf{v}}{2}\right) = 0 \\ \nabla\cdot\left(\sigma\mathbf{V} + \dfrac{\rho\mathbf{v}}{2}\right) = 0 \end{cases}$$

Critical velocity

Ellipticity of mass conservation in the hodograph space:

Frisch… PRL (1992) Rica Physica D (2001) Huynh… PRA (2024)

But before: Chaplygin (1902)

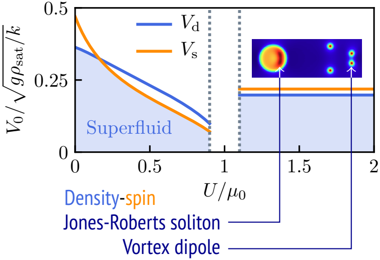
$V < c_{\rm d}(\rho), c_{\rm s}(\rho)$

At least for $U/\mu_0 > 1$ or $u = 0$

Local Landau criterion!

$V_0 < V_{\rm d}(U,u), V_{\rm s}(U,u)$

$g_{+-} = 0.41g$ $U > 0$
$\rho_0 = 3\rho_{\rm sat}$ $u = 0$



Density-spin

Jones-Roberts soliton

Vortex dipole

Superfluid hydrodynamics

$\cos\theta = \sigma/\rho$

Hydraulic: Large w

Momentum

$$\begin{cases} \dfrac{(g+g_{+-})\rho}{1+\rho/\rho_{\mathrm{sat}}} = 2\mu(\rho_0, V_0) - U(\mathbf{r}) - kV^2 - \dfrac{kv^2}{4} \\ \qquad + \dfrac{\nabla^2\sqrt{\rho}}{k\sqrt{\rho}} + \dfrac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\tan\theta} - \dfrac{(\nabla\theta)^2}{4k} \\ \dfrac{(g-g_{+-})\sigma}{1+\rho/\rho_{\mathrm{sat}}} = -u(\mathbf{r}) - k\mathbf{V}\cdot\mathbf{v} - \dfrac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\sin\theta} \end{cases}$$

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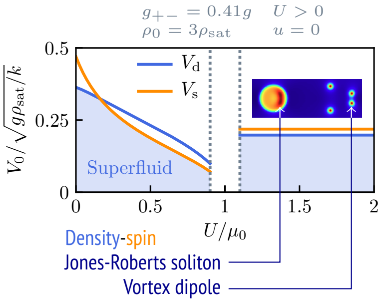

Superfluid hydrodynamics

$\cos\theta = \sigma/\rho$ Hydraulic: Large w Incompressible: Low Mach

Momentum

$$\frac{(g+g_{+-})\rho}{1+\rho/\rho_{\rm sat}} = 2\mu(\rho_0, V_0) - U(\mathbf{r}) - kV^2 - \frac{kv^2}{4} + \frac{\nabla^2\sqrt{\rho}}{k\sqrt{\rho}} + \frac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\tan\theta} - \frac{(\nabla\theta)^2}{4k}$$

$$\frac{(g-g_{+-})\sigma}{1+\rho/\rho_{\rm sat}} = -u(\mathbf{r}) - k\mathbf{V}\cdot\mathbf{v} - \frac{\nabla\cdot(\rho\nabla\theta)}{2k\rho\sin\theta}$$

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$$\nabla\cdot\left(\sigma\mathbf{V} + \frac{\rho\mathbf{v}}{2}\right) = 0$$

$\nabla\cdot\mathbf{V} = \nabla\cdot\mathbf{v} = 0$

Appropriate BC @ $r = w$

Critical velocity

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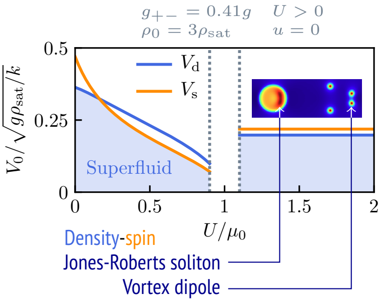
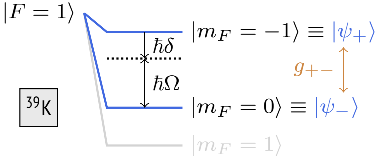


Nonlinear periodic waves in a coherently driven two-component Bose-Einstein condensate

Bose-Einstein condensation in two Rabi-coupled hyperfine states



$$\mathscr{E} = -\frac{\hbar\Omega}{2}\sqrt{\rho^2-\sigma^2} + \frac{\hbar\delta}{2}\sigma + \frac{g+g_{+-}}{4}\rho^2 + \frac{g-g_{+-}}{4}\sigma^2$$

Emergent cubic-quintic nonlinear Schrödinger dynamics

Ground state for $\hbar\Omega \gg |g-g_{+-}|\rho$:

Petrov Bourdel's group Tarruell's group

3-body "interactions"!

$$\mathscr{E}_{\rm gs} \simeq -\frac{\hbar\sqrt{\Omega^2+\delta^2}}{2}\rho + \frac{g_2}{2}\rho^2 + \boxed{\frac{g_3}{3}\rho^3}$$

$$g_2 = g - \frac{g-g_{+-}}{2(1+\delta^2/\Omega^2)}$$

$$g_3 = -\frac{3(g-g_{+-})^2\delta^2/\Omega^2}{4\hbar\Omega(1+\delta^2/\Omega^2)^{5/2}} \boxed{<0}$$

$\Longrightarrow$ Effective cubic-quintic nonlinear Schrödinger theory: $\rho = |\phi(\mathbf{r},t)|^2$

$$i\hbar\partial_t\phi = -\frac{\hbar^2}{2m}\nabla^2\phi + g_2|\phi|^2\phi + g_3|\phi|^4\phi$$

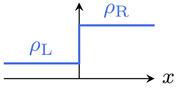
Dispersive shock waves

- 1D regime, natural units:

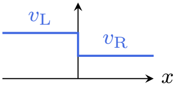
$$i\phi_t = -\tfrac{1}{2}\phi_{xx} + \alpha|\phi|^2\phi + \beta|\phi|^4\phi \qquad \phi(x,t) = \sqrt{\rho(x,t)}\exp[\mathrm{i}\int^x \mathrm{d}x'\, v(x',t)]$$

- Riemann initial-value problem:

$\rho(x, t=0)$

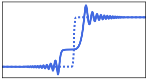


$v(x, t=0)$



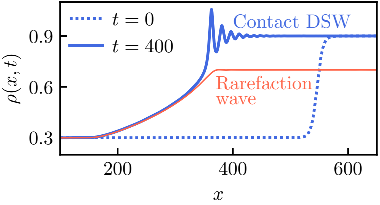
$t > 0$ → Nonlinear evolut°

Typical $\rho(x,t)$ for $\beta = 0$



- Contact dispersive shock waves emerge from quintic nonlinearity!

$\alpha = 1 \quad \beta = -1/2$



Analytics within Whitham modulation theory

Kamchatnov *Nonlinear Periodic Waves and Their Modulations* (2000)

El, Hoefer Physica D (2016)

Within experimental reach @ LCF:

$$(\Delta x, \Delta t)_{\mathrm{cDSW}} \simeq (14.4\,\mu\mathrm{m}, 318.4\,\mathrm{ms})$$

$\omega_\perp/(2\pi) = 300\,\mathrm{Hz} \quad \Omega/(2\pi) = 25.4\,\mathrm{kHz}$

$\rho \simeq 2.5 \times 10^9\,\mathrm{m}^{-1} \quad \delta/\Omega = 0.9$

Summary and future directions

- Conditions for dissipationless motion of a two-component superfluid of light past a polarized obstacle
  First derivation of the critical speed of a 2D two-component nonlinear Schrödinger-type superflow beyond Landau criterion

  ⟶ Nonlinear excitations responsible for the breakdown of superfluidity

  ⟶ Critical velocity for superfluidity beyond the hydraulic regime

- Cubic-quintic nonlinear Schrödinger dynamics in a Rabi-coupled two-component Bose-Einstein condensate
  Contact dispersive shock waves: An experimental signature of nonintegrability due to quintic nonlinearity

  ⟶ Other characteristic nonlinear structures: Anti-dark solitons, kink solitons…

- How to measure the quantum depletion of the fluid of light? Larré, Carusotto PRA (2015)

- Fluctuations of Townes solitons & nonequilibrium Tan contact in a two-component Bose-Einstein condensate

- Emergent Kardar-Parisi-Zhang dynamics in a lossy Bose-Einstein condensate