Wave pattern generated by an obstacle moving in a one-dimensional polariton condensate

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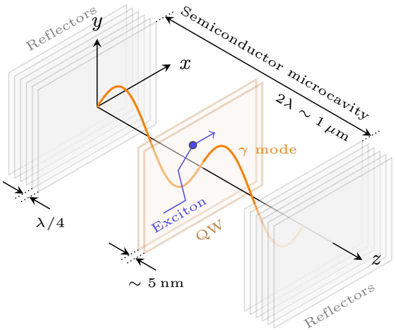
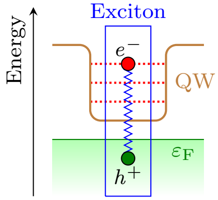
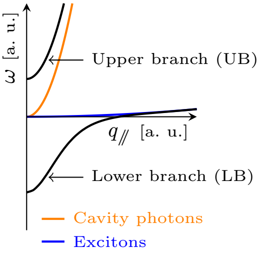
Microcavity polaritons







- ⋆ $\frac{1}{\sqrt{2}}\Big(|\text{Photon}\rangle + |\text{Exciton}\rangle\Big) = |\text{Polariton}\rangle$
- ⋆ Photon, exciton: bosons $\Longrightarrow$ Polariton: boson
- ⋆ Polariton effective mass (LB): $m_{\text{p}}^{*} \lesssim 10^{-4}\, m_{e^-}$
- ⋆ Polariton lifetime: $\tau_{\text{p}} = \tau_{\gamma} \lesssim 50\,\text{ps}$

Polariton condensation

★ Interacting bosons

★ Spontaneous appearance of temporal coherence and long-range spatial coherence

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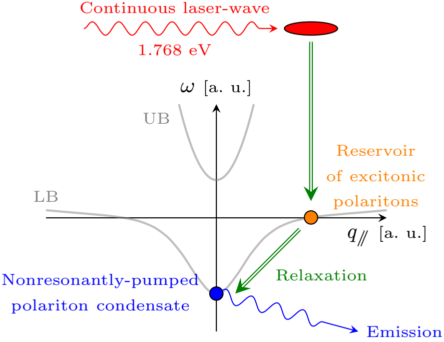
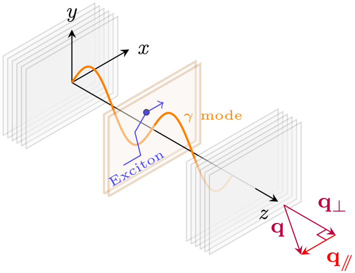
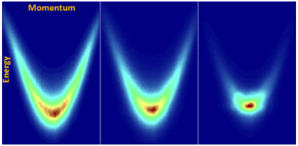
★ Low $m_p^* \Longrightarrow$ High $T_c \sim 10\,\mathrm{K}$

★ Finite polariton-lifetime $\Longrightarrow$ Direct experimental access to internal properties of the polariton fluid just by optical detection of the light emitted by the gas: no intrusive measurements

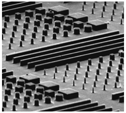






J. Kasprzak *et al.* *Nature* (2006)

★ Grenoble: Institut Néel
★ Lausanne: EPFL ★ . . .

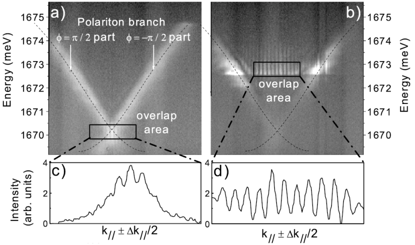


★ Marcoussis: LPN

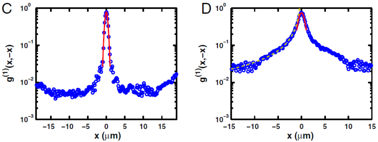
★ Paris: LKB (Jussieu)

Phase coherence in out-of-equilibrium systems

M. Richard *et al.*, *PRL* (2005)



G. Roumpos *et al.*, *PNAS* (2012)



★ $\ell_T \equiv \hbar c_s / T$
★ ρ_s: superfluid density
★ $\lambda_T \equiv \sqrt{2\pi\hbar^2/(mT)}$: thermal wavelength

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★ $\rho_s \lambda^2_{T_{\mathrm{BKT}}} = 4$: vortex/antivortex pairs unbind (Berezinskii–Kosterlitz–Thouless critical point)
★ $\rho_s \lambda_T^2 > 4$: vortex proliferation in a phase with finite-range correlations (condensed phase)

$$\rho^{(1)}_{\mathbf{2D}}(r \equiv |\mathbf{x}-\mathbf{y}|) \equiv \langle \hat{\Psi}^\dagger(\mathbf{x})\, \hat{\Psi}(\mathbf{y}) \rangle$$

$r/\lambda_T \to 0$	$r/\ell_T \to \infty$
$\propto \mathrm{e}^{-\pi r^2/\lambda_T^2}$ (thermal)	$\propto (\ell_T/r)^{1/(\rho_s \lambda_T^2)}$ (condensed)

$$\rho_s \lambda_T^2 \big|_{\mathrm{exp}} \simeq (0.8 - 1.1) < 4$$

Out-of-equilibrium processes: the pumping noise excites phase fluctuations not triggered by vortex proliferation

Superfluidity in polariton condensates

Landau criterion

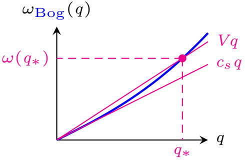
⋆ Weakly perturbing obstacle moving at constant velocity V in a *conservative* quantum fluid at *zero temperature*

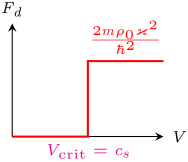
⋆ $\Longrightarrow$ There can exist a critical velocity V_{crit} such that:

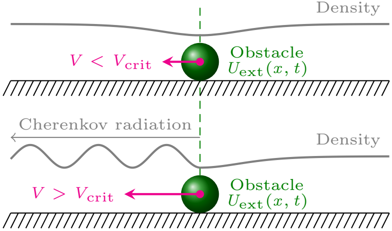
(1) when $V < V_{\text{crit}}$, no excitation is emitted away from the obstacle and there is no drag force: $F_d = 0$ (superfluid regime);

(2) when $V > V_{\text{crit}}$, a Cherenkov radiation of linear waves occurs and the obstacle is subject to a finite drag-force: $F_d \neq 0$ (dissipative regime).

$U_{\text{ext}}(x,t) = \varkappa\,\delta(x + Vt)$ in a quasi-1D BEC

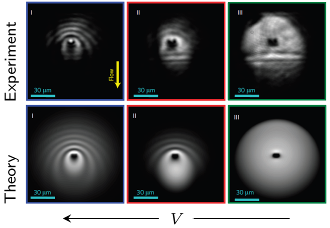






Polariton condensates: nonconservative quantum fluids, but ↴

A. Amo *et al.*, *Nat. Phys.* (2009)



Nonresonantly-pumped polariton condensates at zero temperature: a simple one-dimensional model

Phenomenological modification of the Gross–Pitaevskii equation

$$\boxed{\mathrm{i}\partial_t\psi = -\tfrac{1}{2}\partial_{xx}\psi + U_{\mathrm{ext}}(x,t)\,\psi + \rho\,\psi + \underline{\mathrm{i}\eta\,(1-\rho)\,\psi}}$$

- ⋆ $\psi(x,t)$: condensate wavefunction (scalar because ~~$\sigma = \pm 1$~~)
- ⋆ $\rho(x,t) = |\psi(x,t)|^2$: longitudinal density
- ⋆ $U_{\mathrm{ext}}(x,t)$: potential of an external obstacle

$\partial_t\psi = \eta\,\psi$	$\eta \equiv$ (Gains due to pumping) $-$ (Losses $\propto 1/\tau_{\mathrm{p}}$) > 0
$\partial_t\psi = -\eta\,\|\psi\|^2\,\psi$	Gain saturation
$\Longrightarrow \partial_t\psi\|_{\mathrm{tot}} = \eta\,(1-\|\psi\|^2)\,\psi$	Dynamical equilibrium between gains and losses
	$\Longrightarrow$ Steady-state configuration with $\|\psi_0\|^2 = 1 < \infty$

Finite-size obstacle moving at constant velocity $-M\hat{\mathbf{x}}$ ($M \equiv V/c_s > 0$):

$$U_{\mathrm{ext}} = U_{\mathrm{ext}}(X \equiv x + Mt) \xrightarrow{|X|\to\infty} 0$$

Uniform and stationary solution in the absence of external obstacle:

$$\psi_0(x,t) = \mathrm{e}^{-\mathrm{i}t}, \quad \rho_0(x,t) = |\psi_0(x,t)|^2 = 1$$

Flow past a weakly perturbing impurity

Linear-response theory

$$\psi(x,t) = [1+\delta\psi(x,t)]\,\mathrm{e}^{-\mathrm{i}t}, \quad |\delta\psi(x,t)| \ll 1$$

$$\underbrace{\delta\rho(X)}_{(2)} = \int_{\mathbb{R}} \frac{\mathrm{d}q}{2\pi}\, \underbrace{\chi(q,-Mq)}_{(3)}\, \underbrace{\mathscr{U}_{\mathrm{ext}}(q)}_{(1)}\, \mathrm{e}^{\mathrm{i}qX}$$

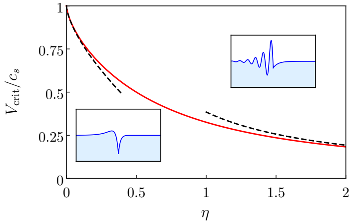
(1) $\mathscr{U}_{\mathrm{ext}}(q) \equiv \int_{\mathbb{R}} \mathrm{d}X\; U_{\mathrm{ext}}(X)\,\mathrm{e}^{-\mathrm{i}qX}$: "source"

(2) $\delta\rho(x,t) \equiv \rho(x,t) - 1$: "response"

(3) Linear-response function

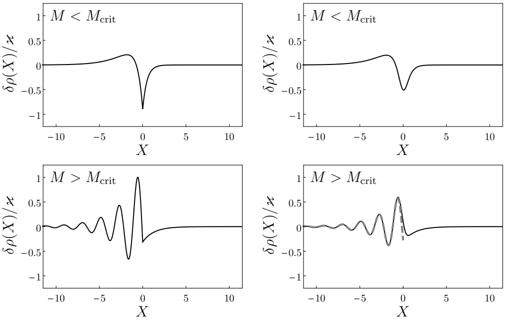
Critical velocity $M_{\mathrm{crit}} \equiv V_{\mathrm{crit}}/c_s$

$$M_{\mathrm{crit}}^2(\eta) = 1 - \frac{3}{2}\,\eta^{\frac{2}{3}} \left(\sqrt[3]{\sqrt{1+\eta^2}+1} - \sqrt[3]{\sqrt{1+\eta^2}-1} \right)$$



$M > M_{\mathrm{crit}}$: emission of a *damped* wake ahead of the obstacle

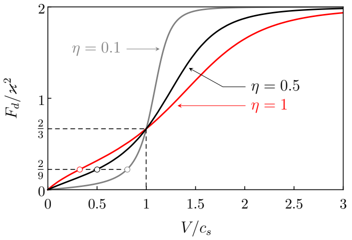
$U_{\mathrm{ext}}(X) = \varkappa\,\delta(X)$ $\qquad$ $U_{\mathrm{ext}}(X) = \dfrac{\varkappa}{\sigma\sqrt{\pi}} \exp\left(-\dfrac{X^2}{\sigma^2}\right)$



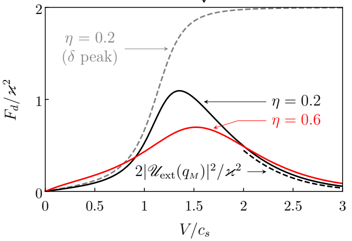
Perturbative drag-force

$$F_d \equiv \int_{\mathbb{R}} \mathrm{d}x\, |\psi(x,t)|^2 \, \frac{\partial U_{\text{ext}}}{\partial x}(x,t) = \begin{pmatrix}\text{drag force experienced} \\ \text{by the obstacle}\end{pmatrix} = -\int_{\mathbb{R}} \mathrm{d}X\, \frac{\mathrm{d}\delta\rho}{\mathrm{d}X}(X)\, U_{\text{ext}}(X)$$

$$U_{\text{ext}}(X) = \varkappa\, \delta(X)$$



$$U_{\text{ext}}(X) = \frac{\varkappa}{\sigma\sqrt{\pi}}\, \mathrm{e}^{-X^2/\sigma^2}$$



★ $F_d|_\delta \overset{M\to 0}{\simeq} \eta M \varkappa^2 \propto M$: "viscous" drag of Stokes type ($\eta \sim$ viscosity)

★ $F_d(M_{\text{crit}})|_\delta = \frac{2}{9}\varkappa^2 =$ ~~fct°(η)~~: onset of (damped) Cherenkov radiations (wave resistance)

★ $F_d|_\delta \overset{M\to\infty}{\simeq} 2\,\varkappa^2 = \begin{cases} \text{~~fct°(η)~~: pure wave-drag} \\ \text{~~fct°(M)~~: } \delta\text{-peak artifact} \end{cases}$

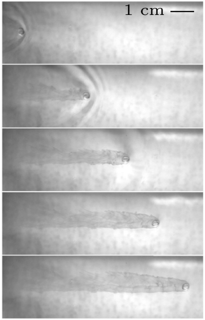
★ $F_d \overset{M\to\infty}{\simeq} 2|\mathscr{U}_{\text{ext}}(q_M)|^2, \quad q_M \equiv 2\sqrt{M^2-1}$

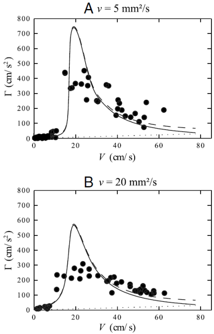
★ For $M > 1$, $F_d|_\delta(\eta) \searrow$ when $\eta \nearrow$: "viscous" effects reduce the range of the wake and diminish the wave resistance which is the dominant source of drag when $M > 1$ (*idem* in the Gaussian case)

★ $F_d|_\delta \overset{\eta\to 0}{\simeq} 2\,\varkappa^2\, \Theta[M - (M_{\text{crit}} \equiv 1)]$: discontinuous behavior in the absence of "viscosity" (well-known result in the atomic-condensation context)

★ $F_d|_{\text{Gaussian}} \xrightarrow{\sigma\to 0} F_d|_\delta$

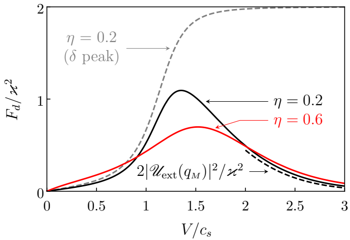
M. Le Merrer *et al.*, *PNAS* (2011)

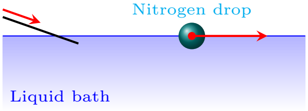




Counterintuitive effect

$$F_d(M > 1) \searrow \quad \text{when} \quad \eta \nearrow$$





Superfluidity?

⋆ Within the framework of our model at zero temperature, a small object moving in a nonresonantly-pumped polariton condensate experiences a finite drag-force at any velocity, revealing a nonsuperfluid behavior of the quantum fluid according to the Landau criterion.

⋆ Similar behavior observed in other related works:
— M. Wouters, I. Carusotto, *PRL* (2010)
— A. Berceanu *et al.*, *J. Phys.* (2012) [resonantly pumped polaritons]

⋆ **However**, the drag-force profile presents a (smooth) crossover between a low-velocity regime and a large-velocity one, recalling the case of nondissipative BECs at $T = 0$,

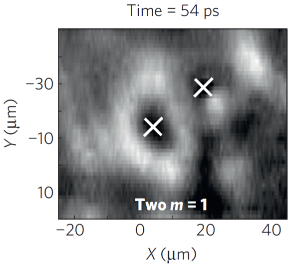
(i) so maybe superfluidity is compatible with "viscous" drag,

(ii) and then maybe $F_d(V)$ is not the best-suited observable to probe superfluidity in dissipative systems.

⋆ After all, $\rho_n \equiv \rho_{\text{tot}} - \rho_s \neq 0, \forall T \in]0, T_\lambda]$ in (superfluid) helium II...

Quantized vortices as a proof of superfluidity in polariton condensates

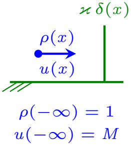
D. Sanvitto *et al.*, *Nat. Phys.* (2010)

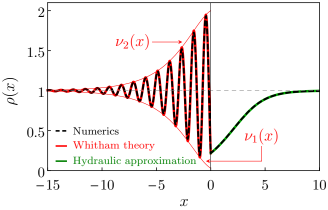


Nonlinear theory for a narrow obstacle

$$\left(\frac{M^2}{2}+1\right)\psi = \left[-\tfrac{1}{2}\partial_{xx} + \varkappa\,\delta(x) + |\psi|^2\right]\psi + \mathrm{i}\eta\,(1-|\psi|^2)\,\psi, \qquad M>1,\quad \eta \ll 1$$

$$\begin{pmatrix} \psi(x) = \sqrt{\rho(x)}\,\mathrm{e}^{\mathrm{i}\theta(x)} \\ \text{with } \theta(x) = \int^x \mathrm{d}x'\,u(x') \end{pmatrix} \Longrightarrow \begin{cases} \partial_x(\rho\,u) = 2\eta\,\rho\,(1-\rho) & \text{(A)} \\ \frac{u^2}{2} + \rho + \frac{(\partial_x\rho)^2}{8\rho^2} - \frac{\partial_{xx}\rho}{4\rho} = \frac{M^2}{2} + 1 & \text{(B)} \end{cases}$$





Hydraulic approx° $(x > 0)$

★ $\partial_x\rho = \mathcal{O}(\eta \ll 1)$: one neglects derivatives of ρ in Eq. (B)

★ Eqs. (A) and (B) $\Longrightarrow$
$$\partial_x\left[\rho\sqrt{M^2 + 2(1-\rho)}\right] = 2\eta\,\rho\,(1-\rho)$$

Whitham modulation theory $(x < 0)$

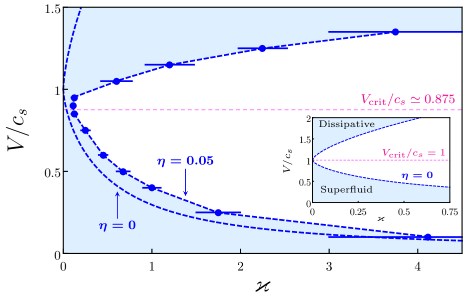
★ $\{\lambda_i(x)\}_{i=1,2,3,4}$: Riemann invariants

★ $\rho(x) = \frac{1}{4}(\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4)^2 + (\lambda_1 - \lambda_2)(\lambda_3 - \lambda_4)$
$$\times\,\mathrm{sn}^2\left[\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\,x,\ \frac{(\lambda_1-\lambda_2)(\lambda_3-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\right]$$

★ $V_\varphi \equiv 0, j, a, L = \text{function}\left(\{\lambda_i(x)\}_i\right)$

★ $\eta \ll 1$: parameters of the dispersive shock-wave vary weakly over one wavelength $\Longrightarrow$ Perturbed Whitham equations:

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}x} = \frac{2}{L}\,\frac{G_1\left(\{\lambda_j\}_j\right)\lambda_i + G_2\left(\{\lambda_j\}_j\right)}{\prod_{j\neq i}(\lambda_i - \lambda_j)}$$
$$= \text{perturbation}_i\left(\{\lambda_j\}_j\right)$$



The two types of steady flows identified in the weak-perturbation limit are separated by a time-dependent regime for strong-enough external potentials, as typically observed in ultracold atomic vapors.

Nonresonantly-pumped spinor polariton condensates

Phenomenological model in 1D

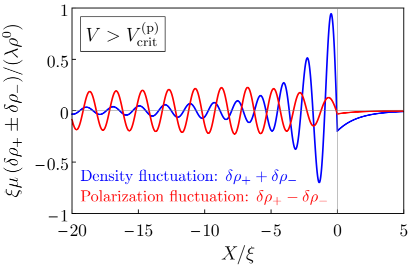
$$\begin{aligned} i\hbar\,\partial_t\psi_\sigma = & -\frac{\hbar^2}{2m}\partial_{xx}\psi_\sigma \\ & + U_{\mathrm{ext}}(x+Vt)\,\psi_\sigma - \sigma\,\hbar\,\Omega\,\psi_\sigma \\ & + (g_1|\psi_\sigma|^2 + g_2|\psi_{-\sigma}|^2)\,\psi_\sigma \\ & + \mathrm{i}\,(\gamma - \Gamma\rho)\,\psi_\sigma \end{aligned}$$

- ⋆ $\sigma = \pm 1$: spin projections onto the z axis
- ⋆ $(\psi_+\ \psi_-)^{\mathrm{T}}$: condensate wavefunction
- ⋆ $\rho(x,t) = |\psi_+|^2 + |\psi_-|^2$: density
- ⋆ $\Omega \propto B_z$: Zeeman splitting between the two polarized states ψ_+ and ψ_-
- ⋆ g_1, g_2: interactions between polaritons with parallel (g_1) and antiparallel (g_2) spins; repulsion dominates: typically,

$$-g_1/10 \sim g_2 < 0 < g_1$$

Linearized theory

- ⋆ Two critical velocities: $V_{\mathrm{crit}}^{(\mathrm{d})} < V_{\mathrm{crit}}^{(\mathrm{p})}$
- ⋆ $V > V_{\mathrm{crit}}^{(\mathrm{d})}$: Cherenkov radiation of *damped* density-waves
- ⋆ $V > V_{\mathrm{crit}}^{(\mathrm{p})}$: Cherenkov radiation of *weakly damped* polarization-waves



P.-É. L., N. Pavloff, A. M. Kamchatnov
In preparation

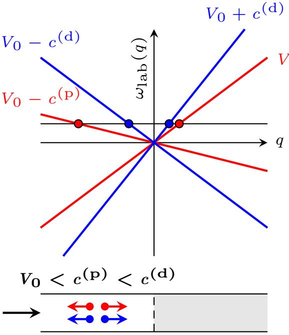
Dumb holes in spinor condensates

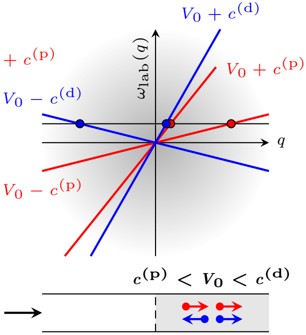
Acoustic horizon for the polarization modes

⋆ $i\hbar\,\partial_t\hat{\Psi}_\sigma = -\frac{\hbar^2}{2m}\partial_{xx}\hat{\Psi}_\sigma + (g_1\,\hat{n}_\sigma + g_2\,\hat{n}_{-\sigma})\,\hat{\Psi}_\sigma \qquad \hat{n}_\sigma(x,t) = \hat{\Psi}^\dagger_\sigma\,\hat{\Psi}_\sigma \qquad 0 < g_2 < g_1$

⋆ Lab-frame dispersion relation of elementary excitations in the long-wavelength limit:

$$\hbar\,\omega_{\text{lab}}(q) \simeq \underset{\text{(Doppler shift)}}{V_0\,\hbar\,q} \pm \begin{pmatrix} c^{(\text{p})} \\ c^{(\text{d})} \end{pmatrix} \hbar\,q \qquad \text{with} \qquad \frac{c^{(\text{p})}}{c^{(\text{d})}} = \sqrt{\frac{g_1 - g_2}{g_1 + g_2}} < 1$$

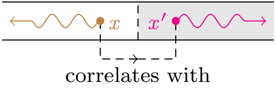




Correlations

$$\hat{\sigma}_z(x,t) = \sum_{\sigma=\pm1} \sigma\,\hat{n}_\sigma(x,t)$$

$$\langle :\hat{\sigma}_z(x,t)\,\hat{\sigma}_z(x',t): \rangle$$



I. Carusotto, S. Finazzi,
P.-É. L., N. Pavloff, A. Recati
In preparation

Wave patterns in polariton condensates: conclusion

⋆ Analyzis of the flow of a one-dimensional scalar polariton condensate in motion with respect to a localized obstacle in a situation of nonresonant pumping at zero temperature

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⋆ **Weak-perturbation limit**: smooth crossover from a viscous flow to a regime where the drag is mainly dominated by wave resistance

⋆ Onset of (damped) Cherenkov radiations at a velocity $V_{\mathrm{crit}}(\eta) \leqslant c_s$ only depending on the parameter η ($\sim$ pumping and losses processes in the system)

⋆ Absence of long-range wake $\neq$ absence of dissipation — Drag force $\neq$ best-suited obervable to probe superfluidity in dissipative condensates?

⋆ Whitham modulation theory and hydraulic approximation in the case of a supersonic fluid flowing past a **δ-peak impurity of arbitrary amplitude**

⋆ The two types of steady flows identified in the weak-perturbation limit are separated by a time-dependent regime for strong-enough external potentials

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⋆ Ejection of a weakly damped polarization-wave: does it make it possible to probe Hawking-like radiation in spinor polariton condensates?

- P.-É. L., N. Pavloff, A. M. Kamchatnov, *Physical Review B* **86**, 165304 (2012)
- P.-É. L., N. Pavloff, A. M. Kamchatnov, *In preparation*
- I. Carusotto, S. Finazzi, P.-É. L., N. Pavloff, A. Recati, *In preparation*